AD/A-005 222

A FLUX CIRCUIT ANALYSIS FOR THE MAGNETIC TRANSDUCER OF A FLUIDIC REED GENERATOR

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Prepared for:

Army Electronics Command

January 1975

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: ECOM-[illegible]

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER: AD/A 005 222

4. TITLE (and Subtitle): A FLUX CIRCUIT ANALYSIS FOR THE MAGNETIC TRANSDUCER OF A FLUIDIC REED GENERATOR

5. TYPE OF REPORT & PERIOD COVERED

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s):
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8. CONTRACT OR GRANT NUMBER(s)

9. PERFORMING ORGANIZATION NAME AND ADDRESS:
Electronic Materials & Nuclear Hardening Rsch Area
US Army Electronics Technology & Devices Lab (ECOM)
Fort Monmouth, NJ 07703 AMSEL-TL-ES

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS:
Proj. No. 1W663613DE5506
Pron No. A9-4-00068-01A9-CT

11. CONTROLLING OFFICE NAME AND ADDRESS:
Harry Diamond Laboratories
Connecticut Ave. at Van Ness NW
Washington, DC 20438

12. REPORT DATE: [illegible]

13. NUMBER OF PAGES: 55

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)

15. SECURITY CLASS. (of this report): Unclassified

15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

Approved for public release; distribution unlimited.

18. SUPPLEMENTARY NOTES

PRICES SUBJECT TO CHANGE

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Artillery Fuzes
Fluidic Generator
Magnetic Permeances
Eddy Current Losses

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A mathematical analysis is made of the magnetic circuit of a fluidic generator to be used in various fuzing applications. A method for obtaining the approximate power output of a prototype generator as a function of the relevant geometric parameters is developed, and sample calculations are made for some typical values of these parameters. In the present design the calculations show that both the vibrating reed and part of the magnet keeper are magnetically saturated under projected operating conditions with the result that only about 35% of the potential power output is actually realized. (Cont'd on reverse side)

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20. Abstract (cont'd)

Experimental data verify this conclusion, as agreement with the calculated value of power is better than 1%. Losses due to eddy currents and hysteresis are found to be much less serious amounting to no more than a few percent. Recommendations of possible remedies for the saturation problem are made. A computer study is made to determine the effect of variation of the relevant design parameters on the peak voltage output of the generator. The resulting design matrices provide a useful guide to design optimization as well as a clear delineation between the parameter combinations which result in saturation of the reed material and those which do not. Recommendations are made for design changes to improve the generator performance.

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A FLUX CIRCUIT ANALYSIS FOR THE MAGNETIC TRANSDUCER
OF A FLUIDIC REED GENERATOR

INTRODUCTION

There is considerable interest in the utilization of fluidic generators in various fuzing applications.[1-3] These devices are mounted in the ogives of various projectiles and they essentially convert some of the energy of the relative motion of projectile and air first into acoustical energy and thence to electrical energy. A typical device is pictured schematically in Fig. 1.

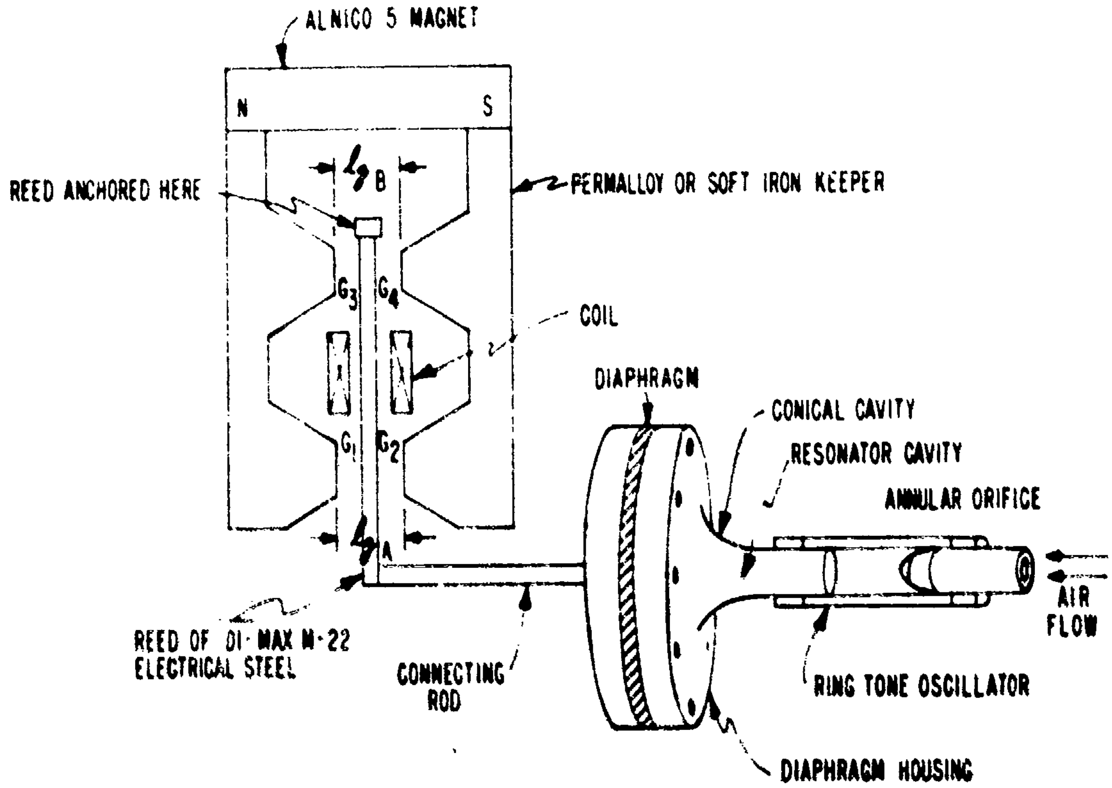

Fig. 1. Schematic View of Fluidic Generator with Reed Type Magnetic Transducer

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1. Carl J. Campagnoulo, "The Fluidic Generator," HDL Technical Report-1328, September 1966.

2. Carl J. Campagnoulo, "Fluidic Power Generators for Ordnance Application," HDL Technical Report-1423, December 1968.

3. Carl J. Campagnoulo and Richard N. Gottron, "The Fluidic Generator: A New Electrical Power Source," 24th Annual Proceedings Power Sources Symposium, May 19-21, 1970.

As the projectile moves forward, air is forced through the annular orifice to excite the resonant cavity beyond. The oscillations of the cavity in turn cause the diaphragm to vibrate. This vibration is then transmitted to a permeable reed of Armco electrical steel. As can be seen from Fig. 1, when the reed is in its equilibrium position it is exactly midway between the pole pieces of both magnetic gaps and, therefore, carries no magnetic flux. The vibration of the reed, however, changes the relative lengths of gaps 1 and 2 and flux passes through the reed, alternating direction with the frequency of vibration. Gaps 3 and 4 are taken to remain constant and equal. The alternating flux through the reed induces an ac voltage in the coil around it and causes a current to flow in a circuit with a load resistance R_L. To optimize the design parameters we need an analysis of the magnetic circuit of the transducer. This will enable us to calculate the energy dissipated in R_L as a function of the gaps between pole pieces (ℓ_g), the reed thickness (t), the reed displacement amplitude (a), and the frequency of vibration of the reed (f).

THE MAGNETIC CIRCUIT EQUATIONS

The magnetic circuit can be replaced by the analogous electric circuit as shown in Fig. 2, where the various electrical admittances have the values

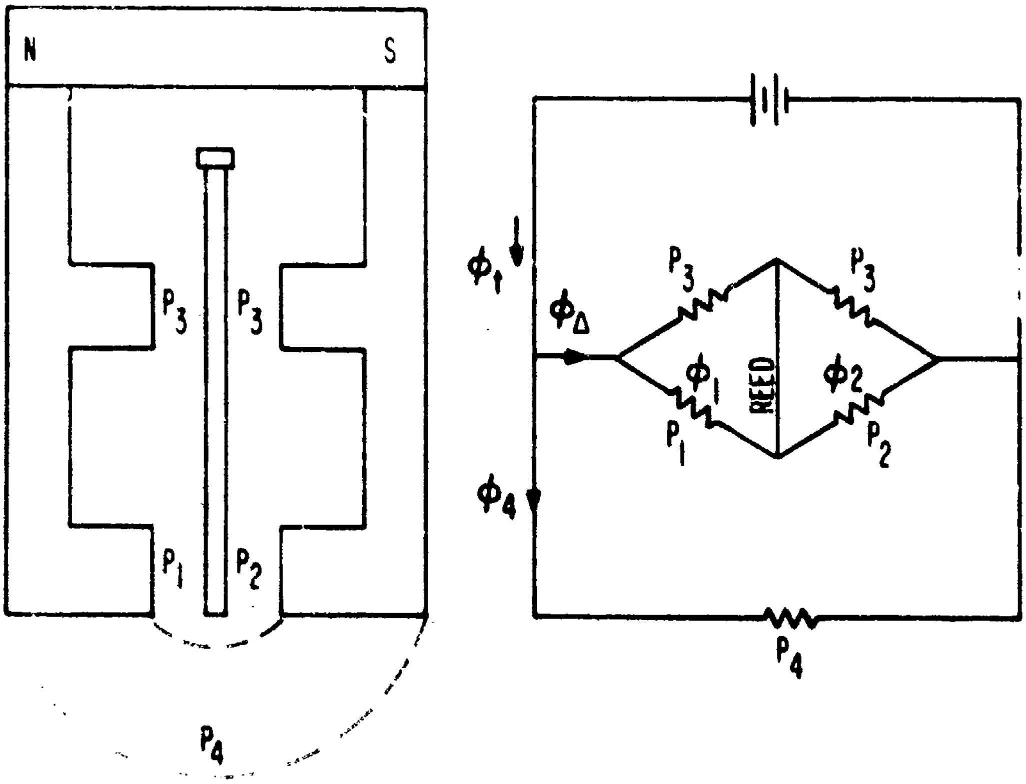

Fig. 2. Schematic Representation of the Magnetic Circuit and its Electrical Analogue

of the relevant magnetic permeances (P) of the reed generator, and the magnitudes of the currents deduced will then be the same as those of the fluxes (Φ) carried in the corresponding branches of the magnetic circuit. The circuit analysis is straightforward:

$$\Phi_\Delta = \frac{P_\Delta}{P_4 + P_\Delta}\Phi_t \tag{1}$$

$$\Phi_1 = \frac{P_1}{P_1 + P_3}\Phi_\Delta \tag{2}$$

$$\Phi_2 = \frac{P_2}{P_2 + P_3}\Phi_\Delta \tag{3}$$

$$\Phi_R = \Phi_1 - \Phi_2 \tag{4}$$

Substituting (1), (2), and (3) in (4) yields

$$\Phi_R = \left(\frac{P_1}{P_1 + P_3} - \frac{P_2}{P_2 + P_3}\right)\frac{\Phi_t}{1 + \frac{P_4}{P_\Delta}} \tag{5}$$

$$\frac{1}{P_\Delta} = \frac{1}{P_1 + P_3} + \frac{1}{P_2 + P_3} \tag{6}$$

$$P_\Delta = \frac{(P_1 + P_3)(P_2 + P_3)}{P_1 + P_2 + 2P_3} \tag{7}$$

Substitution of (7) in (5) finally gives us:

$$\Phi_R = \Phi_t\left(\frac{P_1}{P_1 + P_3} - \frac{P_2}{P_2 + P_3}\right)\frac{1}{1 + \frac{P_4(P_1 + P_2 + 2P_3)}{(P_1 + P_3)(P_2 + P_3)}} \tag{8}$$

$$\Phi_R = \Phi_t\left(\frac{P_1}{P_1 + P_3} - \frac{P_2}{P_2 + P_3}\right)\left(\frac{(P_1 + P_3)(P_2 + P_3)}{(P_1 + P_3)(P_2 + P_3) + P_4(P_1 + P_2 + 2P_3)}\right) \tag{9}$$

$$\Phi_R = \Phi_t\frac{P_1(P_2 + P_3) - P_2(P_1 + P_3)}{(P_1 + P_3)(P_2 + P_3) + P_4(P_1 + P_2 + 2P_3)} \tag{10}$$

$$\Phi_R = \Phi_t\frac{P_3(P_1 - P_2)}{(P_1 + P_3)(P_2 + P_3) + P_4(P_1 + 2P_3 + P_2)} \tag{11}$$

THE PERMEANCE CALCULATIONS

At this point it is necessary to obtain expressions for the various permeances P in terms of the reed displacement amplitude (a) and the configurational parameters which can be conveniently varied, namely the two gap lengths ℓ_{gA} and ℓ_{gB}, and the reed thickness t. Since ℓ_{gA} is always kept equal to ℓ_{gB} we denote both gap lengths by ℓ_g.

These permeances will be calculated by essentially the same techniques used in Ref. 4. These consist principally of approximating the actual flux paths by circular arcs and then applying the permeance formulae for these paths as listed in the standard text by Roters.[5] Suitable modifications of these general formulae must sometimes be made to deal with the peculiarities of a particular configuration. The actual arrangements for the prototype generator in both the isolated and in-shell conditions are shown in Figs. 3 and 4 respectively. The numerical values of the pertinent configurational dimensions are summarized in Table I.

The permeances associated with all of the flux paths which do not pass through the reed are collectively denoted by P4. There are five such paths which are schematically illustrated in Fig. 5, and are calculated immediately below along with the other permeances illustrated in Fig. 5. All calculated permeances are in inches.

(1) <u>P_{p4} (Fig. 5-A)</u>

The flux path here is in the form of a parallel plate magnetic capacitor and the permeance is simply given by

$$P_{P4} = \frac{\text{Plate Area}}{\text{Plate Separation}} = \frac{A_P}{S} = \frac{(.7)(.365)}{.395} = .647'' \tag{12}$$

P_{P4} will be slightly modified by the presence of the reed, but since the reed is very thin compared to the separation of the keeper plates this modification is very small and will be neglected.

(2) <u>P_{y4} (Fig. 5-B)</u>

This is a standard permeance path for which the formula is

$$.318\ y\ \ln\left(1 + \frac{2w}{\ell}\right) \tag{13}$$

where y is the common perimeter length of the keeper edges linked by the flux path. In the fluidic generator y ≈ 2.2 inches. w is the edge thickness .07" and ℓ is the diameter of the smaller of the two semicircular boundaries of the flux path (.395"). Insertion of these values into Eq. (13) yields

4. H.A. Leupold, F. Rothwarf, C.G. Campagnoulo, J. Fine, and H. Lee, "Magnetic Circuit Design Studies for an Inductive Sensor," ECOM Technical Report-4158, October 1973.

5. H. Roters, <u>Electromagnetic Devices</u>, (John Wiley & Sons, Inc., 1941).

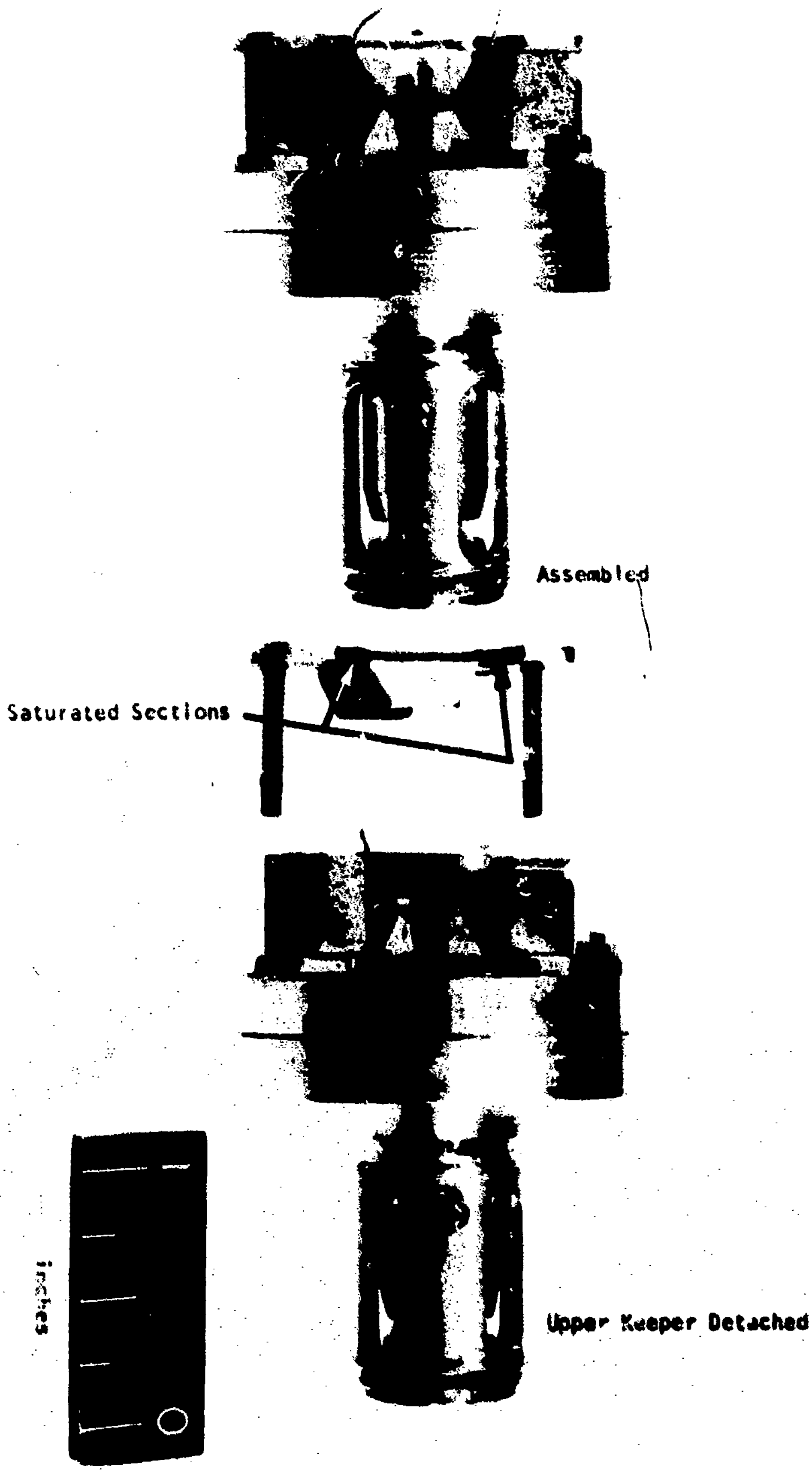

Fig. 3. Fluidic Generator.

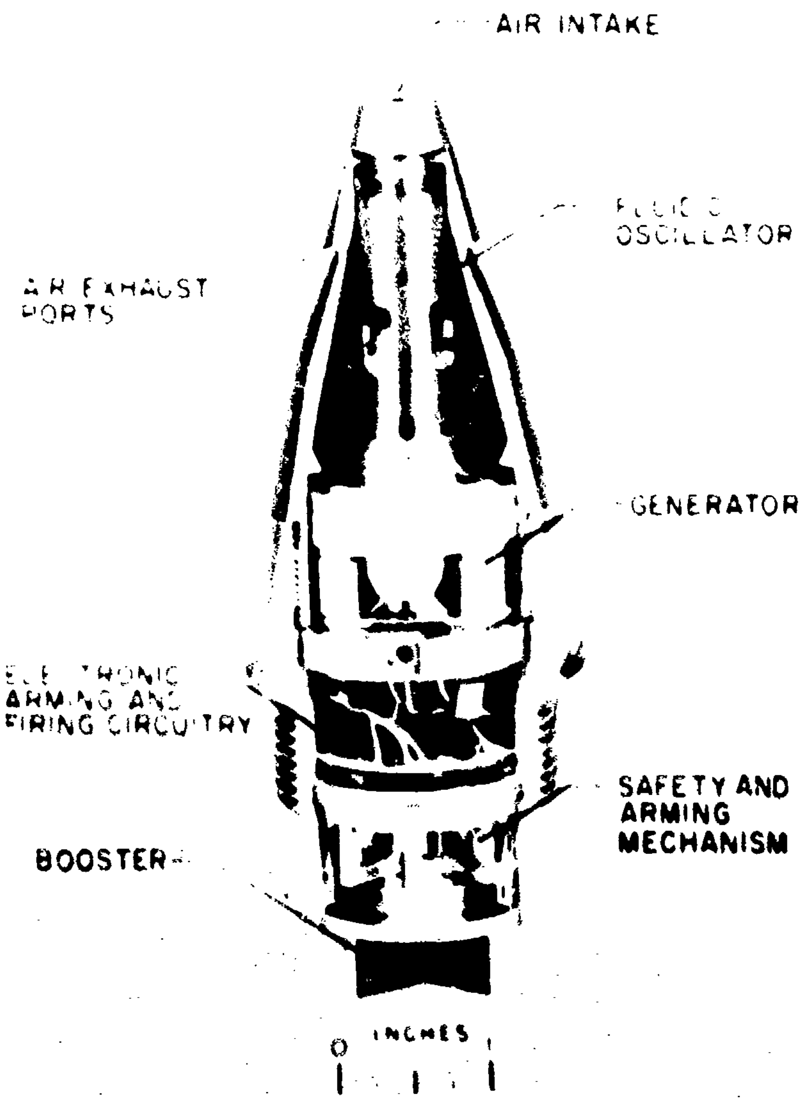

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Fig. 4. Sectional View of Fluidic Generator in Projectile Ogive

TABLE I

Fixed Dimensions of Generator Configuration and Values of Critical Constants

Keeper	Inches	cm
Plate area for P_{μ} calculation	0.256 in^2	1.65 cm^2
Plate separation	0.395	1.00
Thickness	0.070	0.178
Width (narrowest)	0.360	0.914
Perimeter (effective)	2.2	5.6
Saturation flux density	16,000 gauss	
Resistivity ρ	45 $\mu\Omega$ cm	
Hysteresis constant η	0.0001	

Magnet	Inches	cm
Length	0.395	1.00
Cross-section area	0.337 in^2	2.17 cm^2
Effective outer leakage perimeter	2.3	5.8

Poles	Inches	cm
Face thickness	0.070	0.178
Face width	0.313	0.795
Face perimeter	0.766	1.95
Face area	0.0219 in^2	0.141 cm^2
Average width	0.475	1.21
Height (inner)	0.150	0.381
Height (outer)	0.235	0.597
Gap length *	0.070	0.178

Reed	Inches	cm
Width	0.270	0.686
Effective length	0.500	1.27
Thickness *	0.0320	0.0812
Projection beyond gap	0.150	0.381
Vibrational amplitude *	0.017	0.043
Volume *	0.00432 in^3	0.0706 cm^3
Saturation flux density B_m	19,900 gauss	
Resistivity ρ	50 $\mu\Omega$ cm	
Hysteresis constant η	.002	
Projection beyond pole pieces	0.15	0.38

* Values are for sample calculations only. They are variable for computer calculated values in Table II (see Appendix II).

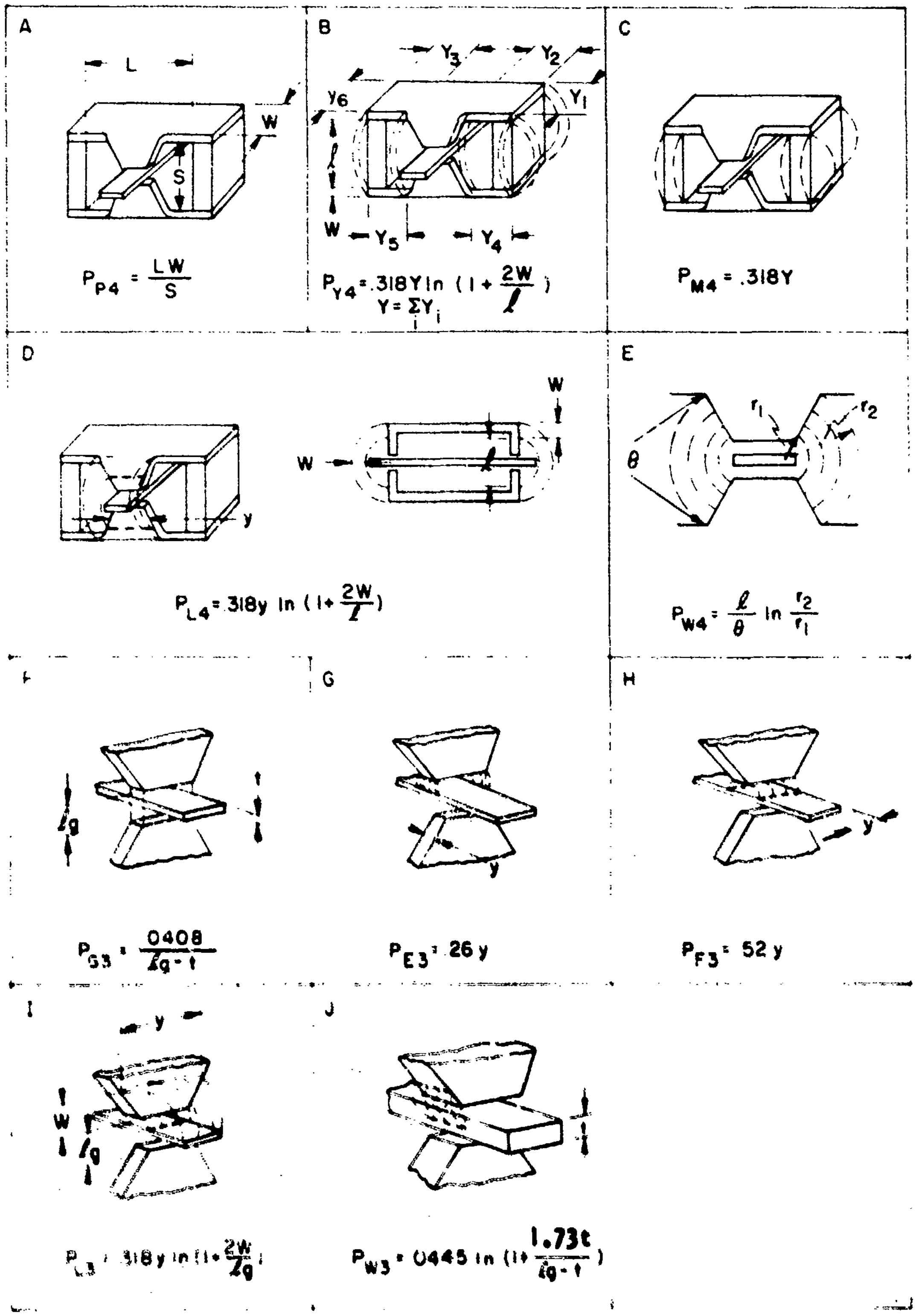

Fig. 5. Calculation of the Relevant Permeances

$$P_{y4} = .318\ (2.2)\ \ln\left(1 + \frac{2(.07)}{.395}\right) = (.318)(2.2)(.303) = .212''$$

(3) P_{M4} (Fig. 5-C)

This is not a standard permeance but an expression for it can be derived as follows: Consider for the time being only that portion of the permeance path formed by the annular ring of width W as shown in Fig. 6. As we shall

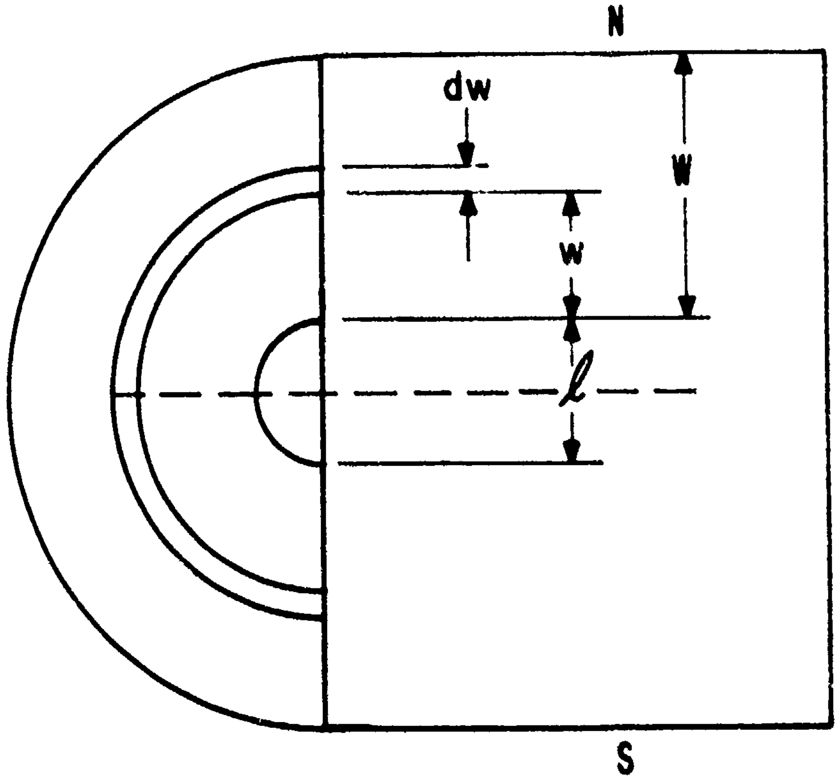

Fig. 6. Calculation of the Permeance P_{M4}

see a simple extension of the resulting formula can then be used to determine the remaining permeance consisting of the semicircular region of diameter ℓ. The permeance of a path such as the annular ring is of the same standard form as Eq. (13), i.e.

$$P = .318\ y\ \ln\left(1 + \frac{2W}{\ell}\right) \tag{14}$$

Similarly, the portion of the permeance within the annular ring of width w is

$$P = .318\ y\ \ln\left(1 + \frac{2w}{\ell}\right) \tag{15}$$

and the permeance of element dw is

$$dP = \frac{.318y \frac{2}{\ell} dw}{1 + \frac{2w}{\ell}} \tag{16}$$

However, since the flux lines are emanating from the sides of a magnet, the magnetomotive force driving an element such as dw is only a fraction of the full MMF of the magnet. Since this MMF is in direct proportion to the distance of the path terminals from the magnet center, this fraction is equal to

$$f = \frac{\frac{\ell}{2} + w}{\frac{\ell}{2} + W} \tag{17}$$

and the effective permeance of dP is reduced by the factor f, viz.,

$$dP_{eff} = fdP = .318y \frac{\left(\frac{\ell}{2} + w\right) \frac{2}{\ell}}{\left(\frac{\ell}{2} + W\right) \left(1 + \frac{2w}{\ell}\right)} dw$$

$$dP_{eff} = \frac{.318y}{\left(\frac{\ell}{2} + W\right)} \tag{18}$$

and the total permeance of the annular ring W is

$$P_{eff} = \frac{.318y}{W + \frac{\ell}{2}} \int_0^W dw = \frac{.318y\, W}{W + \frac{\ell}{2}} \tag{19}$$

To obtain the total effective permeance P_{M4} one needs only to set $\ell = 0$ in the expression for P_{eff} and we are left with

$$P_{M4} = .318y \tag{20}$$

y is just the perimeter of the magnet minus the length of the inner wall. The latter length is subtracted because it is rendered largely ineffective by competition from the flux paths of permeance P_{P4}. The effective y is somewhat awkward to estimate because of irregularities such as the screw slots cut into the magnet walls, but 2.3" seems a reasonable value for the two magnets together. Accordingly, our value for P_{M4} is

$$P_{M4} = (.318)(2.3) = .731''$$

(4) $\underline{P_{L4}\ \text{(Fig. 5-D)}}$

This again is a standard permeance of the form used in P_{y4}. There is one such path for the outside surface of each pole piece so we must multiply Eq. (13) by two. The flux lines emanating from the inner surfaces

go to the reed and hence contribute to P_1, P_2, and P_3 rather than to P_{L4}. For y we take the average width, (.475") of the effective portion of the trapezoidal outer surface of each pole piece. ℓ, (.30") is twice the length of the reed projection beyond the gap as is clear from Fig. 5-D. As .235" is the height of the trapezoidal pole piece plus the gap from the pole face to reed, it also follows from Fig. 5-D that w = (.235 - .150) = .085". Inserting these values in Eq. (13) yields

$$P_{L4} = 2(.318)(.475) \ln\left(1 + \frac{2(.085)}{.300}\right)$$

$$P_{L4} = .302 \quad \ln 1.57 = .136''$$

(5) $\underline{P_{w4}}$ (Fig. 5-E)

The formula for a path of this kind is

$$P = \frac{\ell}{\theta} \ln \frac{r_2}{r_1} \tag{21}$$

and since we have four such paths (2 for each pair of pole pieces)

$$P_{w4} = 4 \frac{\ell}{\theta} \ln \frac{r_2}{r_1} \tag{22}$$

ℓ is just the thickness of our pole pieces which is .070" and θ is $\frac{2}{3}\pi$ radians. r_2 is the distance from the vertex of angle θ to the larger base of the trapezoidal pole piece. Thus, r_2 is equal to the length of the pole piece edge plus one-half the gap width divided by the sine of $\frac{\pi}{3}$ radians. So since the pole piece edge is .2" long

$$r_2 = .2 + \frac{\ell_g}{2} \csc \frac{\pi}{3} = .2 + .577\, \ell_g \tag{23}$$

r_1 is equal to $\frac{\ell_g}{2} \csc \frac{\pi}{3}$ plus the distance along the pole piece edge from which the flux lines go to the edge of the reed. The latter distance is just one half the reed thickness (t). So finally we have

$$r_1 = \frac{t}{2} + .577\, \ell_g \tag{24}$$

and

$$P_{w4} = \frac{(4)(.070)(3)}{2\pi} \ln \frac{.2 + .577\, \ell_g}{\frac{t}{2} + .577\, \ell_g} \tag{25}$$

$$P_{w4} = .134 \ln \frac{.4 + 1.16\, \ell_g}{t + 1.16\, \ell_g} \tag{26}$$

(6) $\underline{P_4}$

We now have expressions for all of the components of the "leakage permeance" P_4, so P_4 is given by:

$$P_4 = P_{P4} + P_{y4} + P_{M4} + P_{L4} + P_{w4}$$

$$P_4 = .647 + .212 + .731 + .136 + .134 \ln \frac{.4 + 1.16\, \ell_g}{t + 1.16\, \ell_g}$$

$$P_4 = 1.72 + .134 \ln \frac{.4 + 1.16\, \ell_g}{t + 1.16\, \ell_g} \qquad (27)$$

If the gap is varied in the usual manner, i.e. by placing permalloy shims between the magnets and the permalloy keeper, P_{w4}, P_{P4}, and P_{y4} will be slightly affected, but since the shim thicknesses are only of the order of 10 or 20 thousandths of an inch the change in these quantities will be negligible.

(7) $\underline{P_{G3}}$ (Fig. 5-F)

This is the permeance of the gap between the reed and either pole piece at the pinned end of the reed. The length of this gap L is given by

$$L = \frac{\ell_g - t}{2} \qquad (28)$$

Since the widths of the pole piece (.313") and the reed .270" are not the same, we take an average (.291") in our determination of the gap's cross-sectional area. The pole face thickness is .070" so

$$P_{G3} = \frac{\text{Area}}{L} = \frac{2(.070)(.291)}{\ell_g - t} \qquad (29)$$

$$P_{G3} = \frac{.0408}{\ell_g - t} \qquad (30)$$

If the gap sizes are varied by machining the pole faces the constant in this formula will change due to the alteration in cross-sectional area of the pole faces.

(8) $\underline{P_{E3}}$ (Fig. 5-G)

P_{E3} is the permeance of the path extending from the edges of the reed to the pole face edges parallel to the former. The standard expression for this type of permeance path is

$$P = .26y \qquad (31)$$

where y is the edge length which here is the thickness of the pole piece (.070") so

$$P = (.26)(.070) = .0182" \quad (32)$$

Since we have two such edges the total permeance is

$$P_{E3} = 2P = (2)(.0182) = .0364" \quad (33)$$

(9) P_{F3} (Fig. 5-H)

These are the paths from the edges of the pole pieces to the flat portion of the reed. These are standard forms whose permeances are given by

$$P = .52y = .52(.291) \quad (34)$$

Since there are two paths we have

$$P_{F3} = 2P = 2(.52)(.291) = .303" \quad (35)$$

(10) P_{L3} (Fig. 5-I)

These permeances involve the paths between the walls of the pole pieces and the wide faces of the reed. The general expressions are the same as those for P_{L4} except for a factor of 2 to take into account that the flux lines here are quarter circles rather than semicircles as in P_{L4}. The gap length ℓ used in the formula must be taken as twice the separation between the reed and the pole piece, i.e.,

$$\ell = 2\left(\frac{\ell_g - t}{2}\right) = \ell_g - t .$$

The quantity w_i for the inner flux path is just the altitude of the trapezoidal pole piece which is .15". For the effective y one chooses an average of the reed width (.270") and the width of the trapezoid midway between its bases (.406"). This results in a value y = .338".

The outer path must be treated slightly differently because the reed projects only .15" beyond the gap edge. Accordingly, w_0 is just taken as

$$w_0 = .15 - \frac{(\ell_g - t)}{2} \quad (36)$$

and the average value y is also slightly altered.

As is seen in Fig. 7 the flux lines from the reed extend only up to line AB which is .15" above the top surface of the reed. The average width of the flux-emanating region of the pole face is then

$$Y_F = \frac{AB + .313}{2} \quad (37)$$

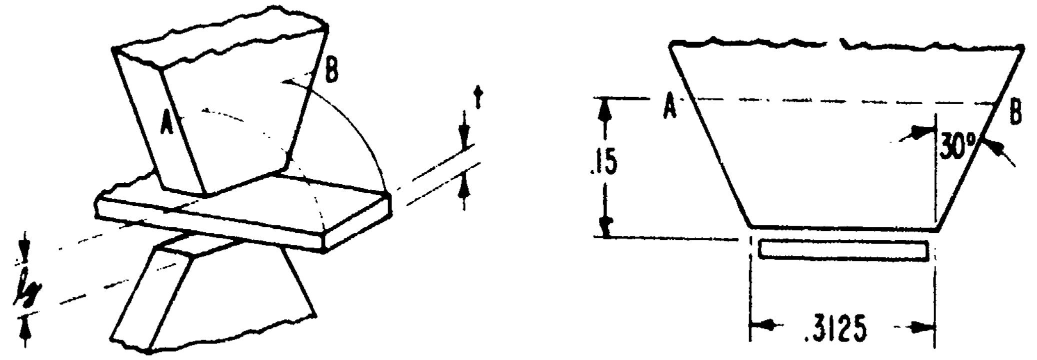

Fig. 7. Determination of the Width y_F in the Formula for Permeance P_{L3}

and

$$y_0 = \frac{y_F + y_R}{2} \tag{38}$$

where y_R = .270 is the reed width.

$$AB = .313 + 2\left(.15 - \left(\frac{\ell_g - t}{2}\right)\right) \tan 30^\circ \tag{39}$$

$$AB = .313 + (.30 - (\ell_g - t))\ .577$$

$$AB = .486 - .577\ (\ell_g - t) \tag{40}$$

$$y_F = \frac{.486 - .577\ (\ell_g - t) + .313}{2} = .399 - .289\ (\ell_g - t) \tag{41}$$

$$y_0 = \frac{.399 - .289\ (\ell_g - t) + .270}{2} \tag{42}$$

$$y_0 = .335 - .144\ (\ell_g - t) \tag{43}$$

Substituting these values in the general expression yields

$$P_{L3} = 2(.318)(.338)\ \ln\left(1 + \frac{2(.15)}{\ell_g - t}\right)$$

$$+ 2(.318)[.335 - .144\ (\ell_g - t)]\ \ln\left(1 + \frac{2\left(.15 - \frac{\ell_g - t}{2}\right)}{\ell_g - t}\right) \tag{44}$$

$$P_{L3} = .215 \ln\left(1 + \frac{.30}{\ell_g - t}\right)$$

$$+ \left(.213 - .0916\,(\ell_g - t)\right) \ln\left(1 + \frac{.30 - (\ell_g - t)}{\ell_g - t}\right) \tag{45}$$

$$P_{L3} = .215 \ln\left(1 + \frac{.30}{\ell_g - t}\right)$$

$$+ \left(.213 - .0916\,(\ell_g - t)\right) \ln\left(\frac{.30}{\ell_g - t}\right) \tag{46}$$

(11) P_{w3} (Fig. 5-J)

If the narrow side face of the reed were coplanar with the narrow face of the pole piece as indicated by the dashed line in Fig. 5-J, the expression for permeance P_{w3} would be the often used standard form

$$P_{w3} = .318y \ln\left(1 + \frac{2w}{\ell}\right) \tag{47}$$

Although it is possible to correct for the fact that the actual flux lines are shorter than in this ideal case, the correction would be very small, and since the total permeance P_{w3} is itself so small as to barely affect the calculations with one significant figure, this correction is completely negligible and will not be made here. So w is taken as $\frac{t}{2}$, $\ell = \frac{\ell_g - t}{2 \cos 30^\circ}$.

$y = .070$, and we precede the expression with the factor 2 because there are two paths of this type

$$P_{w3} = 2(.318)(.070) \ln\left(1 + \frac{2t \cos 30^\circ}{\ell_g - t}\right) \tag{48}$$

$$P_{w3} = .0445 \ln\left(1 + \frac{1.73t}{\ell_g - t}\right) \tag{49}$$

(12) P_{G1} and P_{G2}

These permeances are of the same form as P_{G3} with allowance made for the spacing changes due to the displacement of the reed. When the reed is at equilibrium, that is, at zero displacement, $P_{G1} = P_{G2} = P_{G3}$. Since we are interested in the maximum and minimum values of P_{G1} and P_{G2}, we write them in terms of the displacement amplitude (a) of the reed.

$$P_{G1}^{max} = P_{G2}^{max} = \frac{.0408}{\ell_g - t - 2a} \tag{50}$$

$$P_{G1}^{min} = P_{G2}^{min} = \frac{.0408}{\ell_g - t + 2a} \tag{51}$$

where P_{G1} is a maximum when P_{G2} is a minimum and vice versa.

(13) $\underline{P_{E1}}$

$$P_{E1} = P_{E3} = .0364'' \tag{52}$$

(14) $\underline{P_{F1}}$

$$P_{F1} = P_{F3} = .303'' \tag{53}$$

(15) $\underline{P_{w1} \text{ and } P_{w2}}$

P_{w1} is of the same form as P_{w3} when the reed is in equilibrium. When P_{w1} is a maximum, $\ell = (\ell_g - t)$ must be replaced by $(\ell_g - t - 2a)$ and by $(\ell_g - t + 2a)$ when P_{w1} is a minimum. The same holds for P_{w2}. Thus we have

$$P_{w1}^{max} = P_{w2}^{max} = .0445 \ln\left(1 + \frac{1.73t}{\ell_g - t - 2a}\right) \tag{54}$$

$$P_{w1}^{min} = P_{w2}^{min} = .0445 \ln\left(1 + \frac{1.73t}{\ell_g - t + 2a}\right) \tag{55}$$

(16) $\underline{P_{L1} \text{ and } P_{L2}}$

The term in P_{L1} arising from the inner flux path is the same as the corresponding term in P_{L3} except that again the quantity $(\ell_g - t)$ is replaced by $(\ell_g - t - 2a)$ or $(\ell_g - t + 2a)$ to obtain P_{L1}^{max} and P_{L1}^{min}, respectively.

The term arising from the outer flux path is similarly derived from the corresponding term in P_{L3} by the same substitution but an additional alteration is necessary. The portion of the reed projecting beyond the limits of the gap is triangular rather than rectangular as is the projection at the anchored end of the reed. For this reason the flux lines emanating from the walls of the pole piece have only one half the reed area in which to terminate. To take this into account it seems reasonable to substitute in the expression for y_0, one half of the reed width for the full reed width. Thus, in Eq. (42) the value of .270 is replaced by .135. After all these adjustments are made the expression for P_{L1} and P_{L2} becomes

$$P_{L1}^{\substack{max\\min}} = P_{L2}^{\substack{max\\min}} = .215 \ln\left(1 + \frac{.30}{\ell_g - t \mp 2a}\right)$$

$$+ [.170 - .0916 (\ell_g - t \mp 2a)] \ln \frac{.30}{\ell_g - t \mp 2a} \tag{56}$$

There are other permeances associated with flux paths going from corner to corner, edge to corner and face to corner. In this problem, these are not of standard form and are very difficult to estimate. They tend however, to be small compared to most of the calculated permeances, and the labor of any attempt to estimate them would not be justified by the precision attainable by our computational techniques.

CALCULATIONS OF GAP FLUX DENSITIES IN THE ABSENCE OF A REED

We now have expressions for all of the permeances relevant to the desired power calculations. Before a sample calculation for particular values of ℓ_g, t and a are made, it would be instructive to calculate the field in the gaps when the reed is not present. Since a measured field value exists for a gap width of .060, a comparison with the result of our calculation would give an indication as to the general efficacy of the computational methods used. To do this we need the permeance P_{G4} which is given by:

$$P_{G4} = \frac{(2)(\text{Area Pole Faces})}{\text{Gap Length}} = \frac{(2)(.0219)}{.06} = .730'' \qquad (57)$$

We now also have the permeances P_{E4} between the corresponding edges of the pole faces and P_{c4} between the corresponding corners on either side of the gap. P_{E4} is given by the usual formula $P_{E4} = 2(.26)y$, where y is the perimeter of a pole face and the factor of 2 enters because there are two gaps. P_{c4} is given by $(8)(.077)\ell_g$, where the factor 8 occurs because there are four corner-to-corner permeances for each of the two gaps. Therefore, P_{c4} is given by

$$P_{c4} = (8)(.077)(.060) = .0369'' \qquad (58)$$

$P_{E4} = (.52)(.766) = .398$. P_{P4}, P_{Y4} and P_{M4} are the same as when the reed is present. We, therefore, need only to find the new values of P_{W4} and P_{L4}. P_{W4} is given by Eq. (26) with t set equal to zero. Thus

$$P_{W4} = .134 \ln \frac{.4 + (1.16)(.06)}{1.16(.06)} = .134 \ln 6.75$$

$$P_{W4} = (.134)(1.91) = .256'' \qquad (59)$$

The contributions of the two outer surfaces to P_{L4} are given by the same formula as when the reed is present, but now $\ell = \ell_g = .06$ and

$\bar{y} = \frac{.5 + .313}{2} = .406$. So these contributions

$$P^0_{L4} = 2(.318)(.406) \ln\left(1 + \frac{2(.2)}{.06}\right) = .258 \ln 7.67$$

$$P^0_{L4} = (.258)(2.04) = .526'' \qquad (60)$$

In the absence of the reed the inner surfaces also contribute. The longest semicircular arc of this path, however, must be no longer than the distance d as shown in Fig. 8, so

$$\pi\left(\frac{\ell}{2} + w\right) = d \qquad (61)$$

$$w = \frac{d}{\pi} - \frac{\ell}{2} \qquad (62)$$

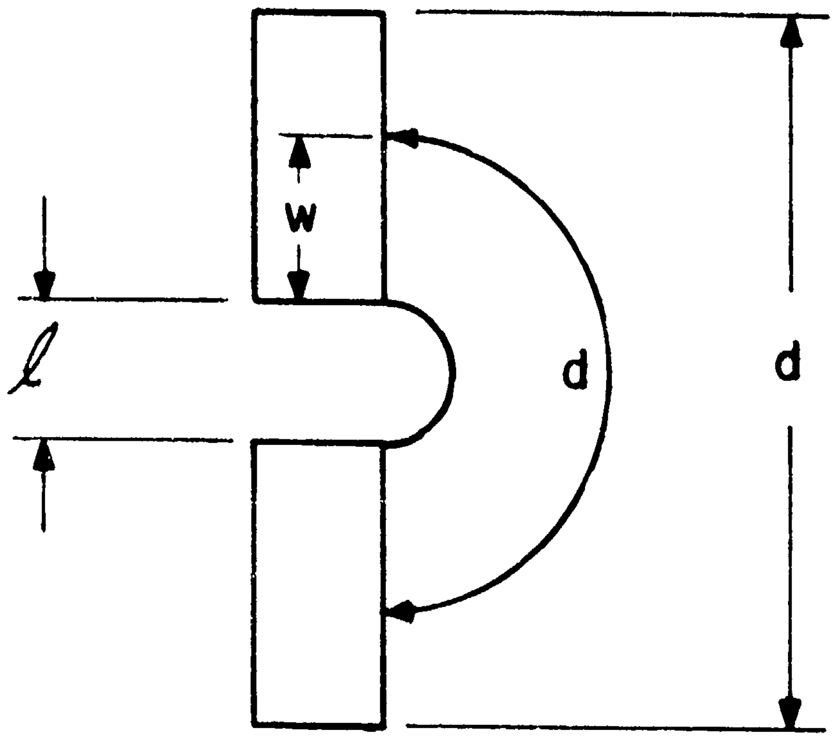

Fig. 8. Determination of the Inner Flux Paths Comprising Permeance P_{M4}

$$w = \frac{.395}{3.14} - \frac{.060}{2}$$

$$w = .126 - .030 = .0958''$$

and

$$\bar{y} = \frac{2(.313) + (2)(.0958)\tan 30^{\circ}}{2} = .313 + .055 = .368'' \tag{63}$$

Thus

$$P^{1}_{L4} = 2(.318)(.368)\ \ln\left(1 + \frac{2(.0908)}{.06}\right) = .234\ \ln\ (4.03) = .325 \tag{64}$$

$$P_{L4} = P^{1}_{L4} + P^{0}_{L4} = .325 + .526 = .851''$$

Our total permeance P_4 in the absence of a reed is then

$$P_4 = P_{G4} + P_{P4} + P_{y4} + P_{M4} + P_{w4} + P_{L4} + P_{E4} + P_{c4} \tag{65}$$

$$P_4 = .730 + .647 + .212 + .731 + .256 + .851 + .398 + .037$$

$$P_4 = 3.86 \tag{66}$$

In this case P_4 is our total permeance P_t and from it we can obtain our load line slope

$$\frac{B}{H} = \frac{L}{A} P_t = \frac{(.395)(3.86)}{(.337)} = 4.52 \tag{67}$$

where L = .395 in and A = .337 in^2. And from the intersection of the Alnico 5 demagnetization curve with the load line B/H = 4.52 in Fig. 9 we obtain

$$\bar{B}_{mag} = 2670 \text{ gauss} \tag{68}$$

Since the magnetic configuration is of irregular shape, B is not perfectly uniform over the entire magnet, therefore, $\bar{B}_{mag}$ represents an effective average flux density. So the total flux output of the magnet is

$$\Phi_t = \bar{B}_{mag} A_{mag} = (2670)(2.17) = 5790 \text{ Maxwells} \tag{69}$$

The flux in one gap Φ_G is given by

$$\Phi_G = \frac{P_{G4}}{2P_t} \Phi_t = \frac{.730}{2(3.86)} 5790 = 547 \text{ Maxwells} \tag{70}$$

and

$$B_G = \frac{\Phi_G}{A_G} = \frac{547}{.141} = 3880 \text{ gauss} \tag{71}$$

This value is about 29% higher than the measured value of 3000 gauss which is satisfactory agreement considering the various approximations made in the course of the calculations. One must also keep in mind that such comparisons should not be taken too seriously. The calculated B_G represents an average flux density obtained by dividing the total flux emanating from a pole face by the area of that pole face. The measured B_G can represent either a much more localized value of B_G or an average that includes fluxes not even associated with the gap, depending on the type, positioning and size of the probe used. The wider the gap compared to the pole face dimensions, the more difficult it is to obtain a meaningful measured value of B_G. Considering these limitations, we conclude that the general computational procedure yields reasonable agreement with the measured results, and that its judicious application to the determination of expected power outputs should not lead us too far astray.

SAMPLE CALCULATION FOR POWER OUTPUT IN THE LOAD RESISTANCE R_L

We will now perform a sample calculation for the most efficient configuration measured thus far. The relevant parameters for this arrangement are: ℓ_g = .070", t = .032", a = .017".

We will first calculate P_4. We need only the components, P_{w4} and P_{L4}. Using Eq. (26) we obtain

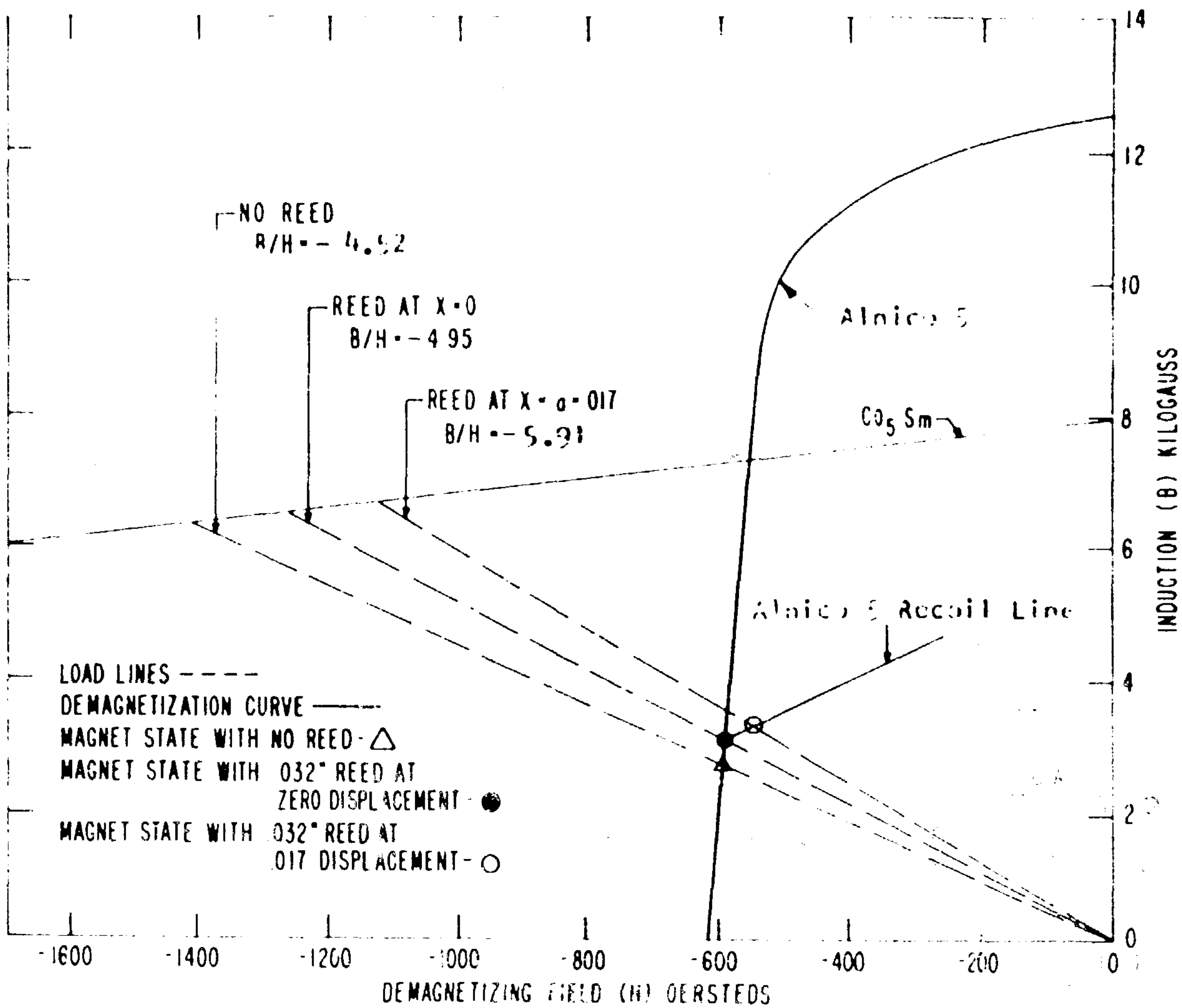

FIG. 9 DETERMINATION OF B FOR A FLUIDIC GENERATOR WITH A .070" GAP LENGTH.

$$P_{w4} = .134 \ln \frac{.4 + 1.16\, \ell_g}{t + 1.16\, \ell_g} = .134 \ln \frac{.4 + (1.16)(.07)}{.032 + (1.16)(.07)}$$

$$P_{w4} = .134 \ln 4.26 = .134(1.45) = .194'' \qquad (72)$$

The appropriate P_{L4} has already been calculated on page 11 and found to be .136. The other components of P_4 are the same as were used to calculate the gap fields except that now there is no P_{G4}, P_{E4} or P_{C4}. Thus

$$P_4 = P_{p4} + P_{y4} + P_{M4} + P_{L4} + P_{w4} \qquad (73)$$

$$P_4 = .647 + .212 + .731 + .136 + .194$$

$$P_4 = 1.92'' \qquad (74)$$

P_3 has five components

$$P_{G3} = \frac{.0408}{.070 - .032} = \frac{.0408}{.038} = 1.07'' \qquad (75)$$

$$P_{E3} = .0364'' \qquad (76)$$

$$P_{F3} = .303'' \qquad (77)$$

$$P_{L3} = .215 \ln \left(1 + \frac{.30}{\ell_g - t}\right)$$

$$+ \left(.213 - .0916\,(\ell_g - t)\right) \ln \left(\frac{.30}{(\ell_g - t)}\right) \qquad (78)$$

$$P_{L3} = .215 \ln \left(1 + \frac{.30}{.038}\right) + (.213 - .0916(.038)) \ln \left(\frac{.30}{.038}\right)$$

$$P_{L3} = .215 \ln 8.90 + (.213 - .0035) \ln 7.90 = (.215)(2.19) + (2.09)(2.07)$$

$$P_{L3} = .470 + .432''$$

$$P_{L3} = .902'' \qquad (79)$$

$$P_{w3} = .0445 \ln \left(1 + \frac{1.73t}{\ell_g - t}\right) = .0445 \ln \left(1 + \frac{(1.73)(.032)}{.038}\right) \qquad (80)$$

$$P_{w3} = .0445 \ln (2.46) = (.0445)(.900) = .0400'' \qquad (81)$$

$$P_3 = 1.07 + .0364 + .303 + .902 + .0400$$

$$\underline{\overline{P_3 = 2.35''}} \qquad (82)$$

P_1 and P_2, when the reed is at its equilibrium position, are obtained by substituting a = 0 in the expression for the components of P_1 and P_2 and then adding these components. This procedure yields $P_1 = P_2 = 2.27''$. We need to find P_t^E, that is, the total permeance at equilibrium to locate the point on the Alnico 5 demagnetization curve at which the recoil line begins. (Point E in Fig. 9)

$$P_t^E = P_4 + \frac{P_3}{2} + \frac{P_1}{2} = 4.23 \tag{83}$$

$$\left(\frac{B}{H}\right)_E = -\frac{\ell}{A} P_t^E = -(1.17)(4.23) = -4.95 \tag{84}$$

$$B_M = 2950 \text{ gauss} \quad \text{(from Fig. 9)} \tag{85}$$

$$H = -\frac{2950}{4.95} = -596 \text{ Oe} \tag{86}$$

The slope of recoil lines for Alnico 5 is 4.3 so the equation of the recoil line is

$$B = 4.3H + B_0 \tag{87}$$

$$2950 = (4.3)(-596) + B_0$$

$$2950 + (4.3)(596) = B_0$$

$$2950 + 2560 = B_0$$

$$5510 = B_0 \tag{88}$$

So we have for our recoil line

$$B = 4.3H + 5510 \text{ gauss} \tag{89}$$

We now need only P_1 and P_2 to complete our analysis. Using Eqs. (50) and (51) we have

$$P_{G1}^{max} = P_{G2}^{max} = \frac{.0408}{\ell_g - t - 2a} = \frac{.0408}{.070 - .032 - 2(.017)} = \frac{.0408}{.004} \tag{90}$$

$$P_{G1}^{max} = P_{G2}^{max} = 10.2'' \tag{91}$$

$$P_{G2}^{min} = P_{G1}^{min} = \frac{.0408}{\ell_g - t + 2a} = \frac{.0408}{.070 - .032 + .034} = \frac{.0408}{.072} = .566'' \tag{92}$$

From (33) and (35) respectively, we have

$$P_{E1} = P_{E2} = .0364'' \tag{93}$$

$$P_{F1} = P_{F2} = .303'' \tag{94}$$

From (54)

$$\left. \begin{matrix} P_{w1}^{max} \\ \\ P_{w2}^{min} \end{matrix} \right| = .0445 \ln\left(1 + \frac{1.73t}{\ell_g - t \mp 2a}\right) = .0445 \ln\left(1 + \frac{(1.73)(.032)}{.070 - .032 \mp 2(.017)}\right)$$

$$P_{w1}^{max} = .0445 \ln\left(1 + \frac{0.0554}{.004}\right) = (.0445) \ln 14.85$$

$$P_{w1}^{max} = (.0445)\ (2.70) = .120'' \tag{95}$$

$$P_{w2}^{min} = .0445 \ln\left(1 + \frac{0.0554}{.072}\right) = .0445 \ln 1.77$$

$$P_{w2}^{min} = (.0445)(.571) = .0254 \tag{96}$$

From (56)

$$P_{L1}^{max} = .215 \ln\left(1 + \frac{.30}{.004}\right) + [.170 - (.0916)(.004)] \ln\left(\frac{.30}{.004}\right)$$

$$P_{L1}^{max} = .215 \ln 76 + .170 \ln 75 = (.215)(4.33) + (.170)(4.32)$$

$$P_{L1}^{max} = .931 + .734 = 1.66 \tag{97}$$

Similarly

$$P_{L2}^{min} = .215 \ln\left(1 + \frac{.30}{.072}\right) + [.170 - (.0916)(.072)] \ln\left(\frac{.30}{.072}\right)$$

$$P_{L2}^{min} = .215 \ln 5.17 + .163 \ln 4.17 = .215(1.64) + (.163)(1.43)$$

$$P_{L2}^{min} = .353 + .233 = .586'' \tag{98}$$

Finally, we have for P_1^{max} and P_2^{min}

$$P_1^{max} = 10.2 + .036 + .303 + .120 + 1.663$$

$$P_1^{max} = 12.32 \tag{99}$$

$$P_2^{min} = .566 + .036 + .303 + .025 + .586$$

$$P_2^{min} = 1.52'' \tag{100}$$

$$P_t^{max} = P_4 + \frac{\left(P_1^{max} + P_3\right)\left(P_2^{min} + P_3\right)}{P_1^{max} + P_2^{min} + 2P_3} \tag{101}$$

$$P_t^{max} = 1.92 + \frac{(12.32 + 2.35)(1.52 + 2.35)}{12.32 + 2.35 + 1.52 + 2.35}$$

$$P_t^{max} = 1.92 + \frac{(14.67)(3.87)}{18.5} = 4.98'' \tag{102}$$

$$\left(\frac{B}{H}\right)^{max} = -\frac{\ell}{A} P_t^{max} = -(1.17)(4.98) = -5.83 \tag{103}$$

From (89)

$$B^{max} = 4.3H + 5510$$

$$B^{max} = \frac{5510}{1 + \frac{4.3}{5.83}} = \frac{5510}{1.73} = 3170 \text{ gauss} \tag{104}$$

$$\Phi_t^{max} = B^{max} A_{magnet} = (3170)(2.18) = 6880 \text{ Maxwells} \tag{105}$$

Substituting all of our calculated permeance values and $\Phi_t^{max} = 6880$ in Eq. (11) yields for the amount of flux carried by the reed

$$\Phi_R^{max} = 6880 \quad \frac{2.35(12.32 - 1.52)}{(14.7)(3.87) + (1.92)(18.5)}$$

$$\Phi_R^{max} = \frac{(6880)(2.35)(10.8)}{92.4} = 1890 \text{ Maxwells} \tag{106}$$

The induced voltage amplitude is then given by

$$V^{max} = \frac{N\omega\,\Phi_R^{max}}{10^8} \tag{107}$$

where ω is the angular frequency of vibration of the reed, which in this case is $2\pi(1500)$ and N = (1500) is the number of turns in the coil.

$$V^{max} = \frac{(1500)(2\pi)(1500)(1890)}{10^8} = 267 \text{ volts} \tag{108}$$

Attached to the coil is a 600-Ω load resistor and the resistance of the coil is 600 Ω as well. The inductance of the coil with a .017" reed is about 23 mhenrys. To find the power dissipated in the load, we need the total impedance Z of the circuit.

$$Z^2 = R^2 + X_L^2 = (1200)^2 + X_L^2 \tag{109}$$

The inductive reactance X_L is just

$$X_L = 2\pi fL = \frac{(6.28)(1500)(23)}{1000} = 217\ \Omega \tag{110}$$

$$Z^2 = (1200)^2 + (217)^2 = 1.49 \times 10^6 \tag{111}$$

$$Z = 1220\ \Omega \tag{112}$$

The root mean square current $\bar{I}$ is given by $\bar{I} = \frac{I_{max}}{\sqrt{2}} = \frac{V_{max}}{Z\sqrt{2}}$ and the power dissipated in the load p_L is then

$$p_L = \bar{I}^2 R_L = \frac{V_{max}^2 R_L}{2Z^2} = \frac{(267)(267)(600)}{(2)(1220)(1220)} = 14.4 \text{ watts} \tag{113}$$

The actual power measured, however, was only 5 watts. The foregoing calculations were made on the assumption that the reed had infinite permeance. The electrical steel of which the reed is composed saturates at 19,900 gauss and its cross-section area is

$$A = (.27 \times .032 \times 2.54 \times 2.54) \text{ cm}^2 \tag{114}$$

Therefore, the maximum amount of flux it can carry is

$$\Phi_s = (19{,}900)(.27)(.032)(2.54)(2.54) = 1110 \text{ Maxwells} \tag{115}$$

and if, as was shown, 1890 Maxwells in the reed would yield 14.4 watts then the maximum allowable flux of 1110 Maxwells would give us

$p_s = \left(\frac{1110}{1890}\right)^2 14.4 = 4.97$ watts which is almost exactly the power observed.

It, therefore, appears that to utilize the full flux output of the magnets used, the reed thickness must be increased by about 70%. Whether this can be done without drastically affecting the vibrational response to the oscillator is a mechanical problem beyond the scope of this study.[3] It is clear, however, that increased output with the present configuration depends largely upon increasing the flux carrying capability of the reed. If, on the other

3. Carl J. Campagnoulo and Richard N. Gottron, "The Fluidic Generator: A New Electrical Power Source," 24th Annual Proceedings Power Sources Symposium, May 19-21, 1970.

hand, the present output of 5 watts is satisfactory, it can be attained with considerably less magnetic material since with the present arrangement 70% more flux is available than can be profitably used. Of course, if the reed thickness is increased, the gaps must be lengthened by an appropriate amount to keep the load line of the magnets the same and to maintain reasonable space in which the reed can oscillate. Also the expected power output would be altered slightly since the inductance of the coil is a function of reed thickness. This effect, however, would not be significant.

Parts of the keeper may also be saturated under the given operating conditions. The cross sections which would saturate first are the ones indicated in Fig. 3. Under the given conditions, the flux that must be carried by the narrowest sections of the keeper is given by

$$\Phi_s = \frac{\Phi_t^{max}}{2} \, \frac{P_t^{max} - P_{m4} - P_{y4}}{P_t^{max}} \tag{116}$$

$$\Phi_s = \frac{6880}{2} \, \frac{4.98 - .731 - .212}{4.98}$$

$$\Phi_s = \frac{6880}{2} \, \frac{4.04}{4.98} = 2790 \text{ Maxwells} \tag{117}$$

and

$$B = \frac{\Phi}{A} = \frac{2790}{(.36)(.07)(2.54)(2.54)} = 17{,}200 \text{ gauss} \tag{118}$$

Since permalloy saturates at 16,000 gauss, the keeper cross section need be increased by only 7% to take full advantage of the available flux at the given reed displacement. This change alone, of course, would do no good unless the reed thickness were doubled as well, as we have already shown.

EXPERIMENTAL CHECK OF CALCULATIONS

To test the validity of the foregoing calculations, a series of measurements was made to determine the maximum voltage output as a function of reed displacement under conditions for which there is no magnetic saturation. The reed displacement was measured by "stopping" the vibrating reed at its maximum displacement (a) with a strobe light and then measuring (a) with a telemicroscope equipped with a calibrated micrometer screw.

The results of this experiment are shown in Fig. 10 where they are compared with the theoretical curve plotted from results of the computer calculations listed in Appendix II. As is apparent from Fig. 10, agreement is excellent considering the many approximations used in the calculations and that (a) can be measured with a precision of no better than ten to fifteen percent.

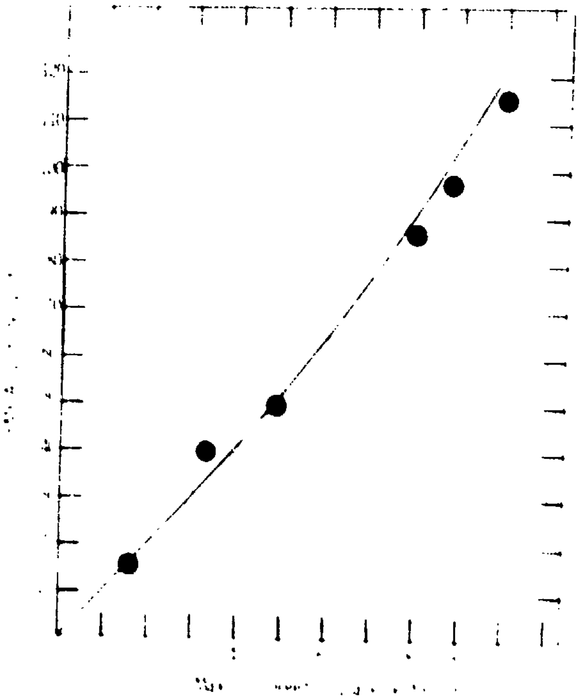

Fig. 10. Comparison of Experimental Points (dots) with Theoretical EMF-Displacement Curve (solid curve)

LOSSES IN THE REED

It is of interest to know how much electrical energy is dissipated in the reed through eddy current losses and hysteresis. The expression for the eddy current power dissipation in watts (see Appendix I) is

$$\frac{P_E}{n} = \frac{\pi^2 f^2 B_m^2 \tau^3 \ell w}{6\rho\ 10^{16}} = \frac{\pi^2 f^2 B_m^2 \tau^2 V}{6\rho\ 10^{16}} \qquad (119)$$

where n is number of laminations, f is the frequency, B_m is the flux density amplitude in gauss, τ is the lamination thickness, ℓ and w are the reed length and width respectively, V is the lamination volume, and ρ is the resistivity in ohm-cm. The reed dimensions are all measured in centimeters and the power is in watts. Since the reed is composed of two laminations each of the thickness τ = .0406 cm, we multiply (119) by two and insert .0406 for τ as well as the appropriate values of the other parameters under saturation conditions.

$$p_E = .34 \text{ watts} \tag{120}$$

The expression for hysteresis power loss in the reed is given by

$$p_H = \eta f^2 \left(\frac{B_m}{10^8}\right)^{1.6} \ell wt = \eta f^2 \left(\frac{B_m}{10^8}\right)^{1.6} V \tag{121}$$

where V = .0706 cc. η is a constant depending on the material and is .002 for our reed. Making the proper insertions gives us

$$p_H = (.002)(1500)(1500)\left(\frac{19.9}{10^5}\right)^{1.6}(.0706)$$

$$p_H = \frac{(.002)(1500)(1500)(1200)(.0706)}{10^{10}}$$

$$p_H = .00004 \text{ watts which is negligible.} \tag{122}$$

The eddy current losses, however, could become quite significant if the reed were not properly laminated. For example, an unlaminated reed of the same thickness as the laminated one would have four times the eddy current loss. It is, therefore, important that if the reed thickness is increased as recommended, it be done by adding laminations. A solid unlaminated reed of twice the thickness of the one presently used would have a power loss of about 11 watts, whereas a fourfold lamination of such a reed would reduce the loss to about .68 watts.

LOSSES IN THE KEEPER

Eddy current losses in the keeper come about because of fluctuations in the flux density as the magnet load line changes with the vibration of the reed. If the keeper thickness is increased by the required 7%, or better still, widened at the narrow points which saturate by 7%, the maximum flux density would be no more than 16,000 gauss. The operating conditions would then be optimized and it would be useful to have some estimate as to the eddy current losses. Such an estimate would be very rough since it is very difficult to determine the flux pattern in the keeper. We already know that when the reed is at its greatest displacement the flux carried by the narrowest portions of the keeper would be 16,000 gauss. We also need the flux density when the reed is in its equilibrium position. This is obtained by substituting the appropriate equilibrium values of the parameters in Eqs. (116 - 118). In this fashion we obtain

$$\Phi_K^E = \frac{\Phi_t^E}{2}\,\frac{P_t^E - P_m 4 - P_y 4}{P_t^E}$$

$$\Phi_K^E = \frac{(2950)(2.17)}{2}\,\frac{4.23 - .731 - .212}{4.23} = 2490 \text{ Maxwells} \tag{123}$$

The cross-section dimensions of the keeper are .914 cm and .178 cm, so

$$B_K^E = \frac{2490}{(.914)(.178)} = 15,300 \text{ gauss}$$

Therefore, the field at the narrowest portion of the keeper fluctuates from 15,300 gauss to 16,000 gauss with an amplitude of $\frac{700}{2} = 350$ gauss. The fluctuation in other parts of the keeper would be somewhat smaller, but 350 gauss can be used to get an upper limit for the expected losses. To this end we use Eq. (119) with the appropriate constants for the keeper whose volume is 2.44 cc and whose thickness would be .178 cm. Further, the fluctuation frequency of B_K^E is double that of the reed since B_K^E is a maximum at both ends of the reed's vibrational cycle. Using these values in Eq. (119) yields:

$$P_E = \frac{(3.14)(3.14)(3000)(3000)(350)(350)(.178)(.178)(2.44)\ 10^6}{(6)(45)10^{16}}$$

$$P_E = .31 \text{ watts} \tag{124}$$

which is about 2% of the expected optimum output. This loss of electrical energy can be avoided by a double lamination of the keeper to cut the loss to 0.08 watts or even a quadruple lamination which would result in the low eddy current loss of 0.02 watts. Since these numbers represent only upper limits, the actual values would probably be much smaller. The use of high-resistivity, high-permeability composite castings for the keepers would probably eliminate the eddy current losses entirely.

The hysteresis loss in the keeper cannot be obtained directly from Eq. (121) because the field in the keeper varies only over positive values of B, while (121) applies only to complete cycles about the principal hysteresis loops. We can, however, get some idea as to the order of magnitude of the loss by substituting our amplitude 350 gauss into (121). Using the value $\eta = .0001$ for permalloy we have

$$P_H = (.0001)(3000)(3000)\left(\frac{350}{10^8}\right)^{1.6}(2.44) \tag{125}$$

$$P_H = 4.1 \times 10^{-6} \text{ watts} \tag{126}$$

which as in the case of the reed is negligible.

SUMMARY AND RECOMMENDATIONS

(1) Under the present conditions of operation the fluidic generator is producing only about 35% of its potential power output. Doubling the cross-section of the reed by adding lamination or increasing its width would augment its flux carrying capability to where the potential power output could be fully realized with only negligible eddy current and hysteresis losses in the reed. Appropriate adjustments in gap length would also be necessary to prevent saturation and to allow sufficient latitude for vibration.

(2) Keeper hysteresis losses are also negligible but eddy current losses can be appreciable unless the keeper is laminated or made of a suitable composite material. Further, the narrowest part of the keeper cross section must be increased by about 7%, to avoid saturation with the given reed displacement amplitude of .017".

(3) To facilitate design optimization of future generators a computer study was done to obtain a series of design matrices which list voltage amplitude as a function of gap length (ℓ_g), reed thickness (τ), and reed displacement amplitude (a). The computer program uses the mathematical procedure of the sample calculation made in this report and it is written in Fortran IV for the Burroughs 5500 Computer. The program is reproduced in Appendix II together with a summary of the equations used in the calculations.

The computed voltage amplitudes are listed in Table II in Appendix II. All of the listed voltages are calculated for a 1500 turn coil, a reed width of .27", a reed vibrational frequency of 1500 hertz, and the standard magnet and keeper dimensions used in the sample calculation and summarized in Table I. The assumption is also made that the keeper does not saturate for any of the parameter choices listed. This assumption, as we have shown in the sample, is not valid for the larger values of (a) unless the keeper thickness is increased by an appropriate amount. This thickening of the keeper would, of course, affect the value of P_{y4} and hence the permeance calculations. The effect on the calculated voltage output, however, is not likely to be significant because in the sample calculation P_{y4} is only of the order of 10% of P_4, and P_4 in turn constitutes only about 40% of P_t. Furthermore, P_{y4} itself is relatively insensitive to keeper thickness (W) since W appears only in a small correction to the logarithmic factor in Eq. (13). Hence, thickening of the keeper to required values will probably not affect the calculated voltages by more than the order of a percent or two even in the worst cases.

The calculations also assume infinite reed permeability and the values not actually attainable with the reed material used are marked with an asterisk. Thus the design can easily be optimized by choosing the parameters yielding the largest non-asterisked voltage in the Tables. This, of course, is always subject to the condition that the reed of requisite thickness can be driven with the stated amplitude and frequency. Perhaps this can be assured in many cases by making appropriate adjustments of the reed stiffness through variation of the reed material and dimensions at the point where it is anchored or by widening the reed rather than thickening it.

The Tables can be used to obtain the power output in any load resistance R_L from Eq. (126), viz.

$$P_L = \frac{V_{max}^2 R_L}{2Z^2} \qquad (127)$$

where Z is the total circuit impedance obtained from Eqs. (109) and (110). The coil inductance is a function of reed thickness and the empirical curve of the relationship is shown in Fig. 11. As is evident from (111), however, the reactive portion of the impedance is only about a percent and a half of the total, and the variation of reactance with reed thickness will not have a significant effect upon power output at the frequency and resistance used.

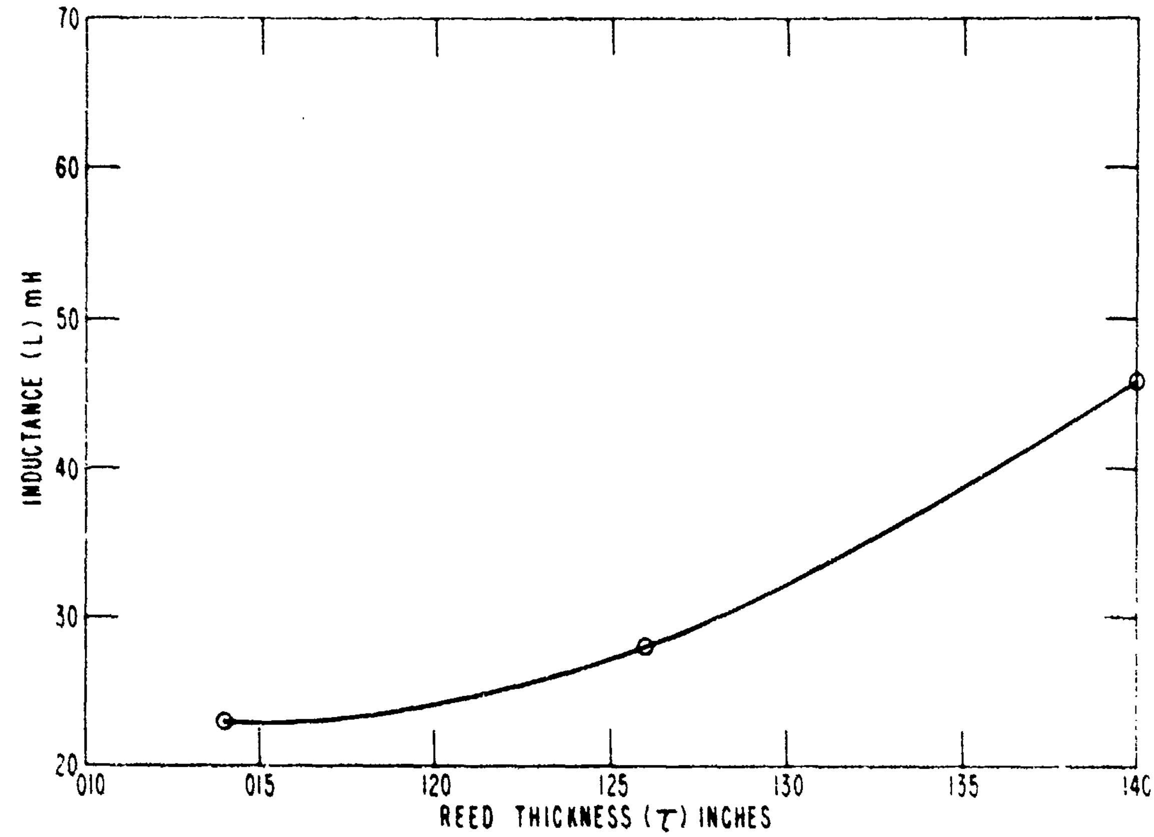

Fig. 11. Inductance of Sensing Coil as a Function of Reed Thickness.

(4) The present configuration of the magnetic circuit results in relatively low-sloped load lines which intersect the Alnico 5 demagnetization curve well below its "knee" (Fig. 9). Note that if Co_5Sm were used in place of Alnico 5 much higher flux densities would result for any given load line with a concomitant increase in power output. This, of course, is always assuming that neither reed nor keeper are saturated. It probably would be most practical to keep the power output at about 10-12 watts with the use of much less magnetic material through substitution of Co_5Sm. A more compact generator operating at lower load lines with the same power output could probably be designed using the latter material.

(5) A composite material consisting of permalloy particles densely packed in a nonconducting binder might be desirable for the keeper. Providing a sufficiently high packing density is obtainable, eddy current losses could be virtually eliminated without too great a loss in flux-carrying capability. Such a composite could also be press molded into the desired shape thus improving the efficiency of fabrication.

APPENDIX I

DERIVATION OF THE FORMULA FOR ENERGY LOSS THROUGH EDDY CURRENTS

Consider a sheet of metal such as the one pictured in Fig. 12. The length ℓ is much larger than the thickness τ and the width w is arbitrary. A sinusoidal field B is acting in the y direction as shown. The varying field B will induce EMFs which will give rise to circulating or eddy currents. We wish to find the power expended by these currents. We proceed by considering a current loop indicated by the arrows in Fig. 12. The EMF around this loop is then given by

$$V = \frac{d\Phi}{10^8 dt} = \frac{A dB}{10^8 dt} = \frac{2\ell x}{10^8} \frac{dB}{dt} \tag{128}$$

If we neglect the small voltage drops across the width of the loop at the ends we can write for the voltage drop per unit length in the z direction

$$v = \frac{V}{2\ell} = \frac{x dB}{10^8 dt} \tag{129}$$

The current density i is then obtained from the relation

$$i = \frac{v}{\rho} \tag{130}$$

where ρ is the resistivity of the material. Substituting (129) in (130) yields

$$i = \frac{x}{10^8 \rho} \frac{dB}{dt} = \frac{x \omega B_{max}}{\rho\ 10^8} \cos \omega t \tag{131}$$

The instantaneous power, dP, dissipated in the unit laminar element dx is then

$$dP = w\rho i^2 dx = \frac{w x^2 \omega^2 B^2_{max} dx}{\rho\ 10^{16}} \cos^2 \omega t$$

$$dP = \frac{w \omega^2 B^2_{max}}{10^{16} \rho} x^2 dx \cos^2 \omega t \tag{132}$$

and the total power loss, P, of a unit length of sheet is

$$P = \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} dP = \frac{w \omega^2 B^2_{max}}{3\rho\ 10^{16}} x^3 \Bigg|_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \cos^2 \omega t \tag{133}$$

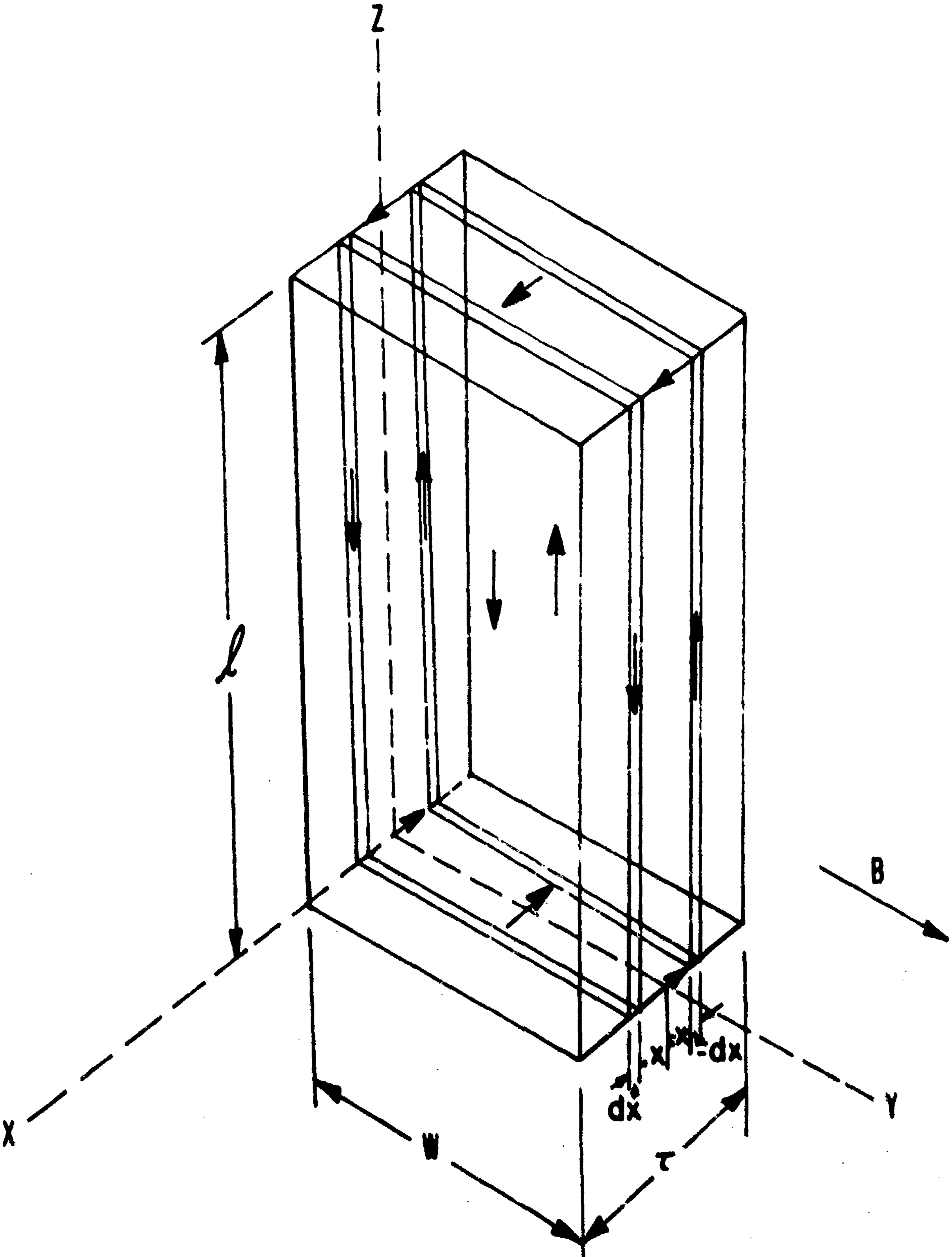

Fig. 12. Derivation of the Eddy Current Loss Formula.

$$P = \frac{w\omega^2 B^2_{max} \tau^3}{3\rho \; 10^{16} 4} \cos^2 \omega t = \frac{w\omega^2 B^2_{max} \tau^3}{12\rho \; 10^{16}} \cos^2 \omega t \qquad (134)$$

The energy loss per cycle is then given by

$$w_c = \frac{w\omega B^2_{max} \tau^3}{12\rho \; 10^{16}} \int_0^{\frac{2\pi}{\omega}} \cos^2 (\omega t)(\omega dt) \qquad (135)$$

$$w_c = \frac{\pi w \omega \tau^3 B^2_{max}}{12\rho \; 10^{16}} \qquad (136)$$

The average power per unit length of sheet is

$$\bar{P} = f w_c = \frac{f \pi \omega \tau^3 w B^2_{max}}{12\rho \; 10^{16}} = \frac{\pi^2 w f^2 \tau^3 B^2_{max}}{6\rho \; 10^{16}} \qquad (137)$$

The average power for the entire length of the sheet is then

$$\bar{P}_t = \ell \bar{P} = \frac{\pi^2 f^2 \tau^2 B^2 \ell w \tau}{6\rho \; 10^{16}} = \frac{\pi^2 f^2 \tau^2 B^2 V}{6\rho \; 10^{16}} \qquad (138$$

where V is the volume of the sheet.

APPENDIX II

SUMMARY OF COMPUTATIONAL PROCEDURE IN CALCULATION OF PEAK VOLTAGE AND ADAPTATION TO COMPUTER PROGRAM

All relevant permeances are calculated for the generator with the reed in the equilibrium position. There are five such permeances of which four are associated with the four air gaps and the fifth with the magnetic leakage paths. The two gaps at the fixed end of the reed are always equal and the associated permeances are equal and denoted by P_3. P_1 and P_2 refer to the gaps at the vibrating end of the reed. P_4 is the permeance of the composite of all leakage paths. P_3 has five components and we can write for the equilibrium condition:

$$P_3 = P_{G3} + P_{E3} + P_{F3} + P_{L3} + P_{W3}$$

all of which are illustrated in Fig. 5 and are computed by the following formulae as functions of ℓ_g, t and a. All other dimensions are fixed and are the same as those of the sample calculations.

$$P_{G3} = \frac{.0408}{\ell_g - t}$$

$$P_{E3} = .0364$$

$$P_{F3} = .303$$

$$P_{L3} = .215 \ln\left(1 + \frac{.30}{\ell_g - t}\right) + [.213 - .0916(\ell_g - t)] \ln\left(\frac{.30}{\ell_g - t}\right)$$

$$P_{W3} = .0445 \ln\left(1 + \frac{1.73\ t}{\ell_g - t}\right)$$

P_4 has five components and is given by

$$P_4 = P_{P4} + P_{y4} + P_{M4} + P_{L4} + P_{W4}$$

and

$$P_{P4} = .647''$$

$$P_{y4} = .212''$$

$$P_{M4} = .731''$$

$$P_{L4} = .136$$

$$P_{W4} = .134 \ln \frac{.4 + 1.16\ \ell_g}{t + 1.16\ \ell_g}$$

P_1 and P_2 have the same form with either always reaching its maximum when the other is at its minimum. Each has 5 components and

$$P_1 = P_{G1} + P_{E1} + P_{F1} + P_{L1} + P_{W1}$$

$$P_{G1\,min}^{\;max} = P_{G2\,min}^{\;max} = \frac{.0408}{\ell_g - t \mp 2a}$$

$$P_{E1} = P_{E2} = .0364''$$

$$P_{F1} = P_{F2} = .303''$$

$$P_{L1\,min}^{\;max} = P_{L2\,min}^{\;max} = .215 \ln\left(1 + \frac{.30}{\ell_g - t \mp 2a}\right)$$

$$+ \left[.170 - .0918(\ell_g - t \mp 2a)\right] \frac{.30}{\ell_g - t \mp 2a}$$

$$P_{W1\,min}^{\;max} = P_{W2\,min}^{\;max} = .0445 \ln\left(1 + \frac{1.73\ t}{\ell_g - t \mp 2a}\right)$$

The total permeance of the magnetic circuit P_t is then given by the formula:

$$P_t = P_4 + \frac{(P_1 + P_3)(P_2 + P_3)}{P_1 + P_2 + 2P_3}$$

and both the equilibrium and full reed displacement values of P_t are calculated. We use the equilibrium value P_t^E to obtain our load line slope B/H via the formula

$$\left(\frac{B}{H}\right)^E = -\frac{\ell}{A} P_t^E$$

where ℓ and A are respectively the length and cross-sectional area of the magnet.

The intersection of this load line with the demagnetization curve of Alnico 5 is the base point of the recoil line (slope 4.3 for Alnico 5) along which the system then operates. The recoil line is thus of the form:

$$B = 4.3\ H + B_o$$

where B_o is determined from the base point. The maximum value of P_t corresponding to full displacement is then used in the relation

$$\left(\frac{B}{H}\right)^{max} = -\frac{\ell}{A} P_t^{max}$$

to obtain the new load line. The intersection of this load line with the recoil line already determined yields the maximum value of B in the magnet.

The corresponding total available flux is then given by

$$\Phi_t^{max} = B^{max} A_{magnet}$$

and the flux amplitude in the reed can then be calculated from the relation

$$\Phi_R^{max} = \Phi_t^{max} \frac{P_3(P_1 - P_2)}{(P_1 + P_3)(P_2 + P_3) + P_4(P_1 + 2P_3 + P_2)}$$

The induced voltage amplitude is then obtained from

$$V^{max} = \frac{N\omega \, \Phi_R^{max}}{10^8}$$

The computer program which follows this procedure is reproduced below in Fortran IV. In addition to calculating the expected voltage for a given set of parameters, the program also instructs the computer to calculate

$$B_R^{max} = \frac{\Phi_R^{max}}{10^8}$$

If the result is larger than the saturation flux density of the reed material (19,900 gauss), the calculated voltage is not attainable and is marked with an asterisk in Table 1.

COMPUTER PROGRAM USED TO OBTAIN TABLE II

```
FILE  1=FILE1,UNIT=PRINTER
   REAL MAX,PW4,PM4,PP4,PY4,PL4,P4,PG3,PF3,PF3,PL3,PW3,P3,PTF,
*       K,M,P,PL,LG
   DIMENSION H(6),R(6),P(3),F(3),VOLT(35),BRFED(35),ASTK(35)
   DATA  (R(I),I=1,6)/1000,8000,9000, 10000,11000,12000/,
*        (H(I),I=1,6)/-617.,-563.,-556.,-520.,-817.,-200./
   Z=1210.
   RL=600.
   TURNS=1500.
   OMEGA=3000*3.1415926
   DO 100 LG=.04,.081,.005
   WRITE(1,500) LG
 500 FORMAT("1",15(1X,/),1X,"GENERATOR OUTPUT IN VOLTS AC FOR GAP "
*"LENGTH = "F5.3,2X,"INCHES",//)
   WRITE(1,97)
 97 FORMAT(7X,"SPEED",3X,/,1X,"THICKNESS",15X,
*      "AMPLITUDE IN INCHES",/,1X,"(INCHES)",1X,".001.002.003.004"
* ".005.006.007.008.009.010.011.012.013.014.015.016.017.018.019"
* ".020.021.022.023.024.025.026.027.028.029 ",/)
   DO 110 T=.01,LG-.005,.002
   N=0
   XN=(LG-T)/2.-.001
   IF(XN.GT.0.0291) XN=0.029
   DO 120 X=.001,XN,.001
   N=N+1
   ASTK(N)=" "
   PW4=.133**ALOG((.4+1.155*LG)/(X+1.155*LG))
   PM4=.7310
   PP4=.4458
   PY4=.2120
   PL4=.1350
   P4=PW4+PM4+PP4+PY4+PL4
   Z=LG-X
   PG3=.0078/Z
   PF3=.0366
   PF3=.3020
   PP=.215*ALOG(1.+.30/Z)
   C=(.2128-.0916*Z)*ALOG(.30/Z)
   PL3=PP+C
   PW3=.0482*ALOG(1.+(1.732*T)/Z)
   P3=PG3+PF3+PF3+PL3+PW3
   F(1)=LG-X-2.*A
   F(2)=LG-X+2.*A
   F(3)=LG-X
   DO 35 I=1,3
   G=.3393+.04452*ALOG(1.+(1.732*T)/F(I))+.04078/F(I)
   B=.215*ALOG(1.+.3/F(I))
   P(I)=G+B+(.1699+.0916*F(I))*ALOG(.3/F(I))
 35 CONTINUE
   PTE=P4+P1/2.+P(3)/2.
   BDHE= -1.172*PTF
   K=ABS(BDHE)
   IF(K.LT.14.18) GO TO 5
   IF(K.GE.14.18 .AND. K.LT.16.19) GO TO 10
   IF(K.GE.16.19 .AND. K.LT.19.23) GO TO 15
   IF(K.GE.19.23 .AND. K.LT.26.38) GO TO 20
   IF(K.GE.26.38 .AND. K.LT.40.) GO TO 25
   I=7
   GO TO 50
 5 I=2
   GO TO 50
 10 I=3
```

```
      GO TO 50
   15 I=4
      GO TO 50
   20 I=5
      GO TO 50
   25 I=6
   50 W=(P(I)-P(I-1))/(H(I)-H(I-1))
      PWE=(P(I)-W*H(I))/(1.-W/ROHE)
      ROE=ROHE-4.3*(PWE/ROHE)
C*       P(1)=P1, P(2)=P2, P(3)=P3 WHEN A=0
      D=(P(1)+P3)*(P(2)+P3)
      E=P(1)+P(2)+2.*P3
      ROHMAX=R*.172*(P4*D/E)
      PMAX=P3/(1.-4.3/ROHMAX)
      FLUXTM=2.17*PMAX
      FLUXRM=FLUXTM*P3*(P(1)-P(2))/(D*P4*E)
      VMAX=TJRNSORWERA*FLUXRM/1.E*P
      PL=VMAX*VMAX*PL/(2*Z*7)
      PREF(N)=FLUXRM/(1.742*T)
      MAX= 19900.
      IF(PREF(N).GT.MAX) ASTK(N)='**'
      VELY(N)=VMAX
  120 CONTINUE
      WRITE(1,13)T,((VELY(I),ASTK(I)),I=1,N)
   13 FORMAT(6X,F6.3,3X,31(I3,A1))
  300 FORMAT(F11.2)
  110 CONTINUE
  100 CONTINUE
      STOP
      END
```

TABLE II

Voltage outputs for various gap lengths, reed thicknesses and reed displacement amplitudes. The values marked with an asterisk are not attainable with the reed material presently in use. They are the values to be expected with the use of reed material that does not saturate under the given operating conditions.

GENERATOR OUTPUT IN VOLTS AC FOR GAP LENGTH = 0.040 INCHES

REED THICKNESS (INCHES)	AMPLITUDE IN INCHES .001	.002	.003	.004	.005	.006	.007	.008	.009	.010	.011	.012	.013	.014	.015	.016	.017	.018	.019	.020	.021	.022	.023	.024	.025	.026	.027	.028	.029
.010	13	26	39	53*	67*	83*	100*	119*	141*	167*	198*	237*	289*																
.012	14	29	44	60*	77*	98*	116*	139*	166*	199*	240*	295*																	
.014	16	33	51	67*	89*	111*	136*	168*	199*	243*	301*																		
.016	19	38	59	80*	104*	131*	162*	199*	246*	308*																			
.018	22	45	69	95*	124*	158*	198*	248*	315*																				
.020	27	54	83	115*	152*	195*	250*	323*																					
.022	32	66	102	143*	190*	250*	330*																						
.024	40	82	128*	182*	248*	337*	469*																						
.026	51	106	168*	243*	363*	493*																							
.028	69	143*	232*	347*	521*																								
.030	87	206*	345*	552*																									
.032	169	326*	586*																										
.034	241*	615*																											

TABLE II (cont'd)

GENERATOR OUTPUT IN VOLTS AC FOR GAP LENGTH = 0.085 INCHES

REED THICKNESS (INCHES) — AMPLITUDE IN INCHES: .001 .002 .003 .004 .005 .006 .007 .008 .009 .010 .011 .012 .013 .014 .015 .016 .017 .018 .019 .020 .021 .022 .023 .024 .025 .026 .027 .028 .029

REED THICKNESS (INCHES)	AMPLITUDE IN INCHES
.010	10 19 29 40 50* 60* 78* 87*101*117*135* [illegible] *129*251*305*
.012	11 22 33 44 54 60* 83* 98*115*134* [illegible] *312*
.014	12 26 37 50 63 78* 98*112*132*155 [illegible]
.016	14 27 41 56 72 89*108*120*153*18[illegible]
.018	15 31 47 62 83 103*125*151*182*210* [illegible] 35*
.020	18 31 55 72 94 120*184*181*221*572* [illegible]
.022	21 42 64 88 110*143*178*221*277*358*
.024	24 40 76 105 137*175*221*281*365*
.026	29 59 92 128*162*220*285*377*
.028	36 73 114 160*214*288*389*
.030	45 93 146 209*290*403*
.032	59 123 196*268*417*
.034	81 170*279*430*
.036	119 255*440*
.038	193*435*
.040	375*

TABLE II (cont'd)

GENERATED OUTPUT IN VOLTS AC FOR GAP LENGTH = 0.050 INCHES

REEL THICKNESS (INCHES)	AMPLITUDE IN INCHES .001	.002	.003	.004	.005	.006	.007	.008	.009	.010	.011	.012	.013	.014	.015	.016	.017	.018	.019	.020	.021	.022	.023	.024	.025	.026	.027	.028	.029
.010	8	15	23	31	39	48	57	67	77	88	100	118	129	144	167	172	223	266	321										
.012	8	17	25	34	43	53	63	74	85	98	112	128	146	168	193	226	269	329											
.014	9	19	28	38	48	58	70	82	95	110	127	146	168	195	229	273	337												
.016	10	21	31	42	53	65	78	92	108	125	145	168	196	232	279	345													
.018	11	23	35	47	60	73	88	105	123	144	168	198	235	284	352														
.020	13	26	39	53	68	83	101	120	142	168	199	238	290	364															
.022	14	29	44	60	77	95	114	139	167	199	241	296	379																
.024	17	33	51	69	89	111	136	165	200	244	302	387																	
.026	19	39	56	81	108	131	162	199	246	309	400																		
.028	22	45	69	96	125	158	198	248	316	415																			
.030	27	54	83	115	152	195	250	323	431																				
.032	32	66	102	143	191	250	330	449																					
.034	40	83	129	172	239	330	470																						
.036	52	106	168	248	348	490																							
.038	69	145	232	387	521																								
.040	87	206	385	552																									
.042	109	327	586																										
.044	261	615																											

TABLE II (cont'd)

GENERATED OUTPUT IN VOLTS AC PER GAP LENGTH = 0.055 INCHES

REEC THICKNESS (INCHES)	AMPLITUDE IN INCHES .001	.002	.003	.004	.005	.006	.007	.008	.009	.010	.011	.012	.013	.014	.015	.016	.017	.018	.019	.020	.021	.022	.023	.024	.025	.026	.027	.028	.029
.010	6	12	19	25	31	38	44	53.	61.	69.	78.	88.	98.	110.	123.	138.	156.	174.	202.	234.	277.								
.012	7	14	20	27	35	42	50	58	67.	76.	86.	97.	109.	123.	138.	156.	177.	208.	237.	282.									
.014	7	15	22	30	38	46	55	64	73.	84.	95.	108.	122.	138.	156.	179.	206.	241.	288.										
.016	8	16	24	33	41	51	60	70	81.	93.	106.	121.	137.	157.	180.	208.	244.	293.											
.018	9	18	27	36	46	56	66	78	90.	104.	119.	137.	157.	181.	210.	248.	299.												
.020	10	20	30	40	51	62	74	87	101.	117.	135.	157.	182.	212.	252.	306.													
.022	11	22	33	44	54	66	81	98	115.	134.	156.	182.	216.	256.	312.														
.024	12	24	37	50	64	78	94	112	132.	155.	183.	216.	260.	320.															
.026	14	27	42	56	72	89	108	129.	154.	183.	218.	264.	328.																
.028	15	31	47	65	83	103	126	151.	182.	220.	269.	336.																	
.030	16	34	55	75	94	121	148.	181.	221.	273.	345.																		
.032	21	42	64	88	114	144	178.	222.	277.	355.																			
.034	[illegible]	49	76	105	137	178.	222.	282.	366.																				
.036	29	60	92	128	170	220.	286.	377.																					
.038	36	73	114	161	217.	289.	390.																						
.040	45	93	146	209.	290.	403.																							
.042	59	123	196	289.	417.																								
.044	81	170	279.	430.																									
.046	119	254.	450.																										
.048	163	435.																											
.050	375.																												

TABLE II (cont'd)

GENERATED OUTPUT IN VOLTS AC FOR GAP LENGTH = 0.040 INCHES

REF. THICKNESS (INCHES)	AMPLITUDE IN INCHES .001	.002	.003	.004	.005	.006	.007	.008	.009	.010	.011	.012	.013	.014	.015	.016	.017	.018	.019	.020	.021	.022	.023	.024	.025	.026	.027	.028	.029
.010	5	10	16	21	26	32	38	44	50	57	63	71	79	87	96	107	118	131	146	163	184	211	245	292					
.012	6	11	17	23	29	35	41	47	54	61	69	77	86	95	106	118	131	146	164	186	213	248	297						
.014	6	12	18	24	31	37	44	51	58	67	75	84	94	105	117	131	147	165	188	216	252	303							
.016	6	13	20	26	33	40	48	56	64	73	82	92	104	114	131	147	166	189	218	256	309								
.018	7	14	21	29	36	44	52	61	70	80	91	102	115	130	147	167	191	221	260	315									
.020	8	15	23	31	40	48	57	67	77	88	101	114	129	147	168	193	224	265	322										
.022	8	17	26	34	44	53	63	74	86	98	113	128	147	168	194	227	269	330											
.024	9	19	28	38	48	59	70	82	96	111	127	146	169	194	230	274	338												
.026	10	21	31	42	53	64	78	93	108	125	145	169	197	229	279	346													
.028	11	23	35	47	60	72	80	105	123	148	149	196	235	285	355														
.030	13	26	39	53	68	84	101	120	142	168	199	238	290	365															
.032	14	29	44	60	77	94	114	140	167	200	241	296	374																
.034	17	33	51	69	90	111	136	165	200	248	303	388																	
.036	19	39	59	81	105	131	162	200	247	308	401																		
.038	22	45	70	96	124	158	188	240	316	415																			
.040	27	54	83	115	152	195	250	323	431																				
.042	32	66	102	143	191	251	331	450																					
.044	40	82	129	167	260	338	470																						
.046	52	106	168	248	349	498																							
.048	69	143	232	348	521																								
.050	97	206	385	551																									
.052	149	327	587																										
.054	241	415																											

TABLE II (cont'd)

GENERATED OUTPUT IN VOLTS AC FOR BAR LENGTH = 0.065 INCHES

REF. THICKNESS (INCHES)	AMPLITUDE IN INCHES .001	.002	.003	.004	.005	.006	.007	.008	.009	.010	.011	.012	.013	.014	.015	.016	.017	.018	.019	.020	.021	.022	.023	.024	.025	.026	.027	.028	.029
.010	4	9	13	18	22	27	32	37	42	47	53*	59*	65*	71*	78*	86*	94*	103*	113*	125*	138*	153*	171*	192*	220*	256*			
.012	5	9	14	19	24	29	34	39	45	51	57	63*	70*	77*	85*	93*	103*	113*	125*	138*	154*	172*	194*	222*	260*				
.014	5	10	15	20	26	31	37	42	48	54	61	68	76*	84*	93*	102*	113*	125*	138*	154*	173*	194*	225*	264*					
.016	5	11	16	22	28	33	39	44	52	59	66	74	82*	91*	101*	112*	125*	139*	155*	174*	198*	228*	269*						
.018	6	12	18	24	30	36	43	49	56	64	72	81	90*	100*	112*	124*	139*	156*	176*	200*	231*	273*							
.020	6	13	19	25	31	38	46	54	61	70	79	88	99*	111*	124*	139*	156*	177*	202*	235*	278*								
.022	7	14	21	28	35	42	50	58	67	76	86	97	110*	123*	139*	157*	178*	206*	238*	283*									
.024	7	15	22	30	38	46	55	64	74	84	94	108	122*	138*	157*	179*	206*	241*	288*										
.026	8	16	24	32	42	51	60	70	81	91	106	121	138*	157*	180*	209*	245*	294*											
.028	9	18	27	36	46	56	67	78	91	108	120	137	157*	181*	211*	249*	300*												
.030	10	20	30	40	51	62	74	87	102	118	136	157*	182*	213*	252*	306*													
.032	11	22	33	45	57	69	83	98	115	134	156	183*	215*	254*	311*														
.034	12	24	37	50	64	78	94	112	132	155	183*	217*	260*	320*															
.036	14	27	42	57	72	89	108	129	154	183*	219*	265*	329*																
.038	15	31	47	65	83	103	126	152	182	220*	269*	337*																	
.040	18	36	55	75	97	121	148	181	221*	273*	346*																		
.042	21	42	64	89	114	144	179	222*	278*	355*																			
.044	24	49	76	105	137	175	222*	282*	366*																				
.046	29	60	92	129	170	220	286*	377*																					
.048	36	73	114	161	217	282*	390*																						
.050	45	93	146	210	290*	405*																							
.052	59	121	196	288*	417*																								
.054	81	170	279*	431*																									
.056	119	254	440*																										
.058	193	439*																											
.060	375*																												

TABLE II (cont'd)

GENERATOR OUTPUT IN VOLTS AC FOR GAP LENGTH = 0.070 INCHES

REED THICKNESS (INCHES) — AMPLITUDE IN INCHES

REED THICKNESS (INCHES)	.001	.002	.003	.004	.005	.006	.007	.008	.009	.010	.011	.012	.013	.014	.015	.016	.017	.018	.019	.020	.021	.022	.023	.024	.025	.026	.027	.028	.029
.010	4	8	11	15	19	23	27	32	36	40	45	50	55	60	66	72	78	85	92	100	109	119	131	144	159	177	200	229	269
.012	4	8	12	15	20	25	29	33	38	43	48	53	59	66	70	77	84	91	100	109	119	131	144	160	179	202	232	273	
.014	4	9	13	17	22	26	31	36	41	46	51	57	63	69	76	83	91	99	109	119	131	145	161	180	204	235	278		
.016	5	9	14	18	23	28	33	38	44	49	55	61	68	74	82	90	99	108	119	131	146	162	182	207	239	283			
.018	5	10	15	20	25	30	35	41	47	53	59	66	71	81	89	98	108	119	132	146	163	184	209	242	287				
.020	5	10	16	21	27	32	38	44	50	57	64	71	79	88	97	107	119	132	147	164	185	211	245	293					
.022	6	11	17	23	29	35	41	48	54	62	69	77	86	96	106	118	132	147	165	187	214	249	298						
.024	6	12	18	25	31	37	44	51	59	67	75	84	94	105	118	131	147	164	188	216	253	304							
.026	7	13	20	27	33	41	48	56	64	73	83	93	104	117	131	147	167	190	219	257	310								
.028	7	14	21	29	36	44	52	61	70	80	91	103	116	130	147	167	192	222	261	316									
.030	8	15	23	31	40	48	58	67	77	89	101	114	130	147	168	193	224	265	321										
.032	8	17	26	34	44	53	63	74	86	99	113	129	147	169	195	227	270	331											
.034	9	19	28	38	48	59	70	83	96	111	127	144	169	194	230	275	338												
.036	10	21	31	42	54	66	79	93	108	126	146	169	197	231	280	347													
.038	11	23	35	47	60	74	89	105	123	144	169	198	236	285	356														
.040	13	26	39	51	68	84	101	120	142	168	199	239	291	366															
.042	15	29	44	60	77	96	115	140	167	200	242	297	377																
.044	17	33	51	69	89	111	136	165	200	244	303	388																	
.046	19	39	59	81	105	132	163	200	247	310	401																		
.048	22	45	70	96	125	158	199	249	317	416																			
.050	27	58	83	118	152	194	250	324	432																				
.052	32	66	102	143	191	251	331	450																					
.054	40	82	129	183	249	339	471																						
.056	52	108	168	244	344	490																							
.058	69	144	232	348	522																								
.060	97	206	365	551																									
.062	149	327	587																										
.064	261	615																											

TABLE II (cont'd)

GENERATOR OUTPUT IN VOLTS AC FOR GAP LENGTH = 0.075 INCHES

REFC THICKNESS (INCHES)	AMPLITUDE IN INCHES .001	.002	.003	.004	.005	.006	.007	.008	.009	.010	.011	.012	.013	.014	.015	.016	.017	.018	.019	.020	.021	.022	.023	.024	.025	.026	.027	.028	.029
.012	3	7	10	13	17	20	24	27	31	35	39	43	47	51*	56*	61*	66*	71*	77*	83*	90*	97*	105*	114*	124*	136*	149*	165*	184*
.013	4	7	11	14	18	21	25	29	33	37	41	45	50	54*	60*	65*	70*	76*	83*	89*	97*	105*	114*	125*	136*	150*	166*	186*	210*
.014	4	7	11	15	19	23	27	31	35	39	44	48	53	58	64	69*	75*	82*	89*	97*	105*	114*	125*	137*	151*	167*	187*	213*	266*
.016	4	8	12	16	20	24	28	33	37	42	47	52	57	62	68	74	81*	88*	96*	105*	114*	125*	137*	152*	169*	189*	215*	250*	
.018	4	8	13	17	21	26	30	35	40	45	50	55	61	67	73	80	87	95*	104*	114*	125*	138*	153*	170*	191*	218*	253*		
.020	4	9	13	18	23	27	32	37	42	48	53	59	65	72	79	86	95	106*	114*	125*	138*	153*	171*	193*	221*	257*			
.022	5	9	14	19	24	29	34	40	45	51	57	63	70	78	85	94	103	116*	125*	139*	154*	172*	195*	223*	261*				
.024	5	10	15	20	26	31	37	43	49	55	62	69	76	84	93	103	113	128*	139*	155*	174*	197*	226*	265*					
.026	5	11	16	22	28	34	40	46	52	59	67	74	83	92	102	113	125	139*	155*	175*	199*	229*	270*						
.028	6	12	18	24	30	36	43	50	57	64	72	81	90	101	112	125	139*	156*	176*	201*	232*	274*							
.030	6	13	19	26	32	39	46	54	62	70	79	89	99	111	124	139	157*	177*	203*	235*	279*								
.032	7	14	21	28	35	42	50	59	67	77	87	98	110	123	139	157*	179*	205*	238*	284*									
.034	7	15	22	30	38	46	55	64	74	84	96	108	123	139	157	180*	207*	242*	289*										
.036	8	16	24	33	42	51	60	71	82	94	107	121	138	157	181*	209*	245*	295*											
.038	9	18	27	36	46	56	67	78	91	105	120	137	157	181	211*	249*	301*												
.040	10	20	30	40	51	62	74	87	102	118	136	157	182	213*	253*	307*													
.042	11	22	33	45	57	70	83	99	115	134	157	183	215*	257*	310*														
.044	12	24	37	50	64	79	95	112	132	154	183	217*	261*	321*															
.046	14	27	42	57	72	90	108	130	154	183	219	265*	329*																
.048	15	31	48	65	83	103	126	152	183	220	269*	337*																	
.050	18	36	55	75	97	121	149	181	222	274*	346*																		
.052	21	42	64	88	114	144	179	222	278*	356*																			
.054	24	49	76	105	137	176	222	282*	366*																				
.056	29	60	92	128	170	221	286*	378*																					
.058	36	73	114	161	217	289*	390*																						
.060	45	93	146	210	290	404*																							
.062	59	123	196	288	417*																								
.064	81	170	280	431*																									
.066	119	256	440*																										
.068	103	435*																											
.070	375*																												

TABLE II (cont'd)

GENERATOR OUTPUT IN VOLTS -- FOR GAP LENGTH = 0.080 INCHES

SEED THICKNESS (INCHES)	AMPLITUDE IN INCHES .001	.002	.003	.004	.005	.006	.007	.008	.009	.010	.011	.012	.013	.014	.015	.016	.017	.018	.019	.020	.021	.022	.023	.024	.025	.026	.027	.028	.029
.010	3	6	9	12	15	18	21	24	27	31	34	37	41	45	49	53*	57*	61*	66*	71*	76*	82*	88*	95*	102*	110*	119*	129*	141*
.012	3	6	9	12	16	19	22	25	29	32	36	40	43	47	51	56	60*	65*	70*	75*	81*	88*	94*	102*	110*	119*	130*	141*	155*
.014	3	7	10	13	[illegible]	20	23	27	30	34	38	42	46	50	55	59	64	69*	75*	81*	87*	94*	102*	110*	119*	130*	142*	156*	174*
.016	3	7	10	14	[illegible]	21	25	28	32	36	40	44	49	53	58	63	68	74	80*	87*	94*	101*	110*	120*	130*	143*	158*	175*	197*
.018	4	7	11	15	18	22	26	30	34	38	43	47	52	57	62	67	73	79	86	93*	101*	110*	120*	131*	144*	159*	177*	199*	227*
.020	4	8	12	15	[illegible]	23	28	32	36	41	45	50	55	60	66	72	78	85	93	101*	110*	120*	131*	144*	160*	178*	201*	230*	270*
.022	4	8	12	16	[illegible]	25	29	34	38	43	48	53	59	65	71	77	84	92	100	109*	120*	131*	145*	161*	180*	203*	233*	273*	
.024	4	9	13	17	[illegible]	26	31	36	41	46	51	57	63	69	76	83	91	100	109	120*	132*	145*	162*	181*	205*	236*	279*		
.026	5	9	14	19	[illegible]	28	33	38	44	49	55	61	68	75	82	90	99	109	120	132*	146*	163*	183*	207*	239*	283*			
.028	5	10	15	20	[illegible]	30	36	41	47	53	59	66	73	81	89	98	108	119	132	147*	164*	184*	210*	243*	288*				
.030	5	11	16	21	[illegible]	32	38	44	50	57	64	71	79	88	97	108	119	132	147	165*	186*	212*	246*	293*					
.032	6	[illegible]	17	23	[illegible]	35	41	48	55	62	69	78	86	96	107	119	132	147	165*	187*	214*	250*	299*						
.034	6	[illegible]	18	25	[illegible]	38	44	52	59	67	76	85	95	106	118	132	148	166	189*	217*	250*	305*							
.036	7	[illegible]	20	27	[illegible]	41	48	56	63	73	83	93	108	117	131	148	167	190*	219*	258*	311*								
.038	7	14	21	29	[illegible]	44	53	61	70	80	91	103	116	131	148	166	192*	222*	262*	317*									
.040	8	[illegible]	23	32	[illegible]	49	58	67	78	89	101	115	130	148	168	193	225*	256*	324*										
.042	8	[illegible]	26	35	[illegible]	53	64	74	86	99	113	129	147	169	195	228*	271*	331*											
.044	9	19	28	39	[illegible]	60	70	83	96	111	128	147	169	194	230*	275*	339*												
.046	10	[illegible]	31	42	[illegible]	66	79	93	108	126	146	169	198	233*	280*	347*													
.048	11	23	35	47	[illegible]	70	89	105	123	144	169	199	236*	286*	356*														
.050	13	26	39	53	[illegible]	84	101	120	143	168	200	239	291*	366*															
.052	15	29	[illegible]	60	77	96	117	140	167	200	242	297*	377*																
.054	17	[illegible]	[illegible]	70	[illegible]	112	136	165	200	245	303*	389*																	
.056	19	39	59	81	[illegible]	132	163	200	247	310*	402*																		
.058	22	45	70	94	[illegible]	159	199	249	[illegible]	416*																			
.060	27	54	[illegible]	114	152	196	251	320*	432*																				
.062	32	66	102	143	191	251	331*	450*																					
.064	40	82	129	183	249	338*	471*																						
.066	52	106	168	244	334*	495*																							
.068	69	144	232	348*	522*																								
.070	97	206	345*	553*																									
.072	149	327	587*																										
.074	241	614*																											